TEXAS A&M UNIVERSITY
DEPARTMENT OF MATHEMATICS

MATH 425-500

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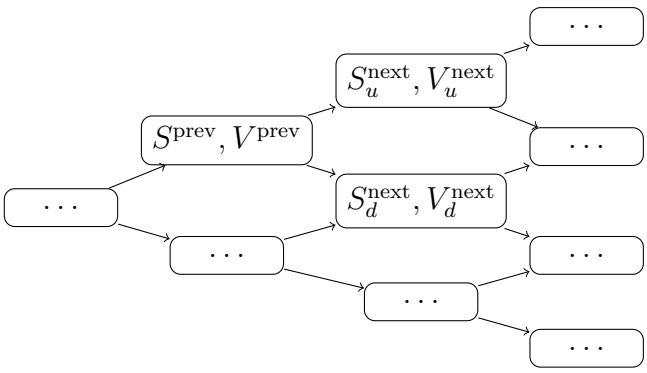
On my honor, as an Aggie, I have neither given nor received unauthorized aid on this work.

Name (print):

- You **must** show work. No work no points no kidding no cry.
- Each question is worth 10 points, except the binomial tree calculation and hedging, which is worth 20.

1. Draw the profit diagram for the following spread (called "iron butterfly"): buy one $E = 30$ put for \$0.75, buy one $E = 50$ call for \$0.65, write one $E = 40$ put for \$3.12, write one $E = 40$ call for \$2.88. In what region of prices S_T will you make a profit?
You may use 2 pages for your answer.

2. When using binomial trees to value American options, see figure below,



we assume $S_u^{\text{next}} = uS^{\text{prev}}$, $S_d^{\text{next}} = dS^{\text{prev}}$ and use $V^{\text{prev}} = \max(\text{EE}, \widetilde{V})$, where EE is the value that would be obtained by early exercise and

$$\widetilde{V} = e^{-r\Delta t}\Big(V_u^{\text{next}} q + V_d^{\text{next}}(1-q)\Big), \qquad \text{with } q = \frac{e^{r\Delta t} - d}{u - d}.$$

1. Verify the "matringale property" of q:

$$S_u^{\text{next}} q + S_d^{\text{next}}(1-q) = S^{\text{prev}} e^{r\Delta t}.$$

2. Prove the following rule: "when applying binomial tree method to American Put, if (early) exercise was optimal when calculating both V_d^{next} and V_u^{next}, it will also be optimal when calculating V^{prev}."

You may use 2 pages for your answer. (Part 2 of this question requires a bit of thinking to express the "rule" in mathematical terms. You may want to leave it for the last.)

3. (20 points) Price the following *American put option* using the tree model: stock price now is $S_0 = 100$, strike is $E = 115$, interest rate is 10%, time to expiration 4 months.

1. Construct the **4-level** tree (1 level per month) if the stock price is expected to either go up by 10% or go down by 10%. Price the above option using this tree.
2. Describe the hedging procedure undertaken by a writer of the option who seeks to eliminate risk. You must assume that the holder of the option will do what is best for them (i.e. exercise early if optimal). Consider the price path: Up, Down, Up, Down.

You may use 2 pages for your answer. (Hint: you may use the "rule" from the previous question to speed up your calculation)

4. Let $X_t = X(t, W_t)$ satisfy the stochastic equation

$$dX = 0.03X dt + 0.2X dW.$$

Use Ito chain rule

$$df\big(t, g(t, W_t)\big) = \frac{\partial f}{\partial t} dt + \frac{1}{2}\frac{\partial^2 f}{\partial g^2}(dg_t)^2 + \frac{\partial f}{\partial g} dg_t,$$

to find the stochastic equation satisfied by $Y_t = 1/X_t$. (There shouldn't be any X left in your equation for Y)

Points:	**/50**