Betting systems: how not to lose your money gambling

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Gambling and Games of Chance

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  - flipping a coin (head / tail)
  - rolling a die (6 sided)
  - roulette (18 black, 18 red and 2 green)
- Player vs casino
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Fair Game

Examples:

- Coin tossing with wager 1$ —
- Roulette red-black, wager 1$ —
- Die: 5 or above win 2$, 4 or below lose 1$ —
- Die: 5 or above win 5$, 4 or below lose 3$ —
- 2 dice: sum 8 or above win 20$, sum 7 or below lose 15$ — ?

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$$\text{payment} = \begin{cases} \text{positive if a win,} \\ \text{negative if a loss.} \end{cases}$$

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  - average $= 1 \times \frac{1}{2} + (-1) \times \frac{1}{2} = 0$
- Roulette
  - #total = 18 + 18 + 2 = 38.
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Another type of average

- Previous average was the *average winning* in one round.
- Another important average is *average capital after round N*:
- Example:
  - you had 10$ and played a round of coin tossing.
  - if you won (probability 1/2), you have 11$.
  - if you lost (probability 1/2), you have 9$.
  - on average you have $9 \times \frac{1}{2} + 11 \times \frac{1}{2} = 10$ dollars.
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- Previous average was the *average winning* in one round.
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Gambler's Ruin : The Problem

- Initially you have n dollars.
- You play a game with a wager of 1$ in each round.
- In each round, the probability to win is p.
- $p = 1/2$ is fair, $p < 1/2$ is subfair.
- You stop when you get to m dollars or lose all your money.
- What is the probability to win the day?
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- Probability theory grew out of this problem.
- And it provides the answer:

$$P(n \to m) = \begin{cases} \frac{\rho^n - 1}{\rho^m - 1} & \text{if } \rho \neq 1, \\ \frac{n}{m} & \text{if } \rho = 1, \end{cases}$$

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Betting Systems: Definition

- Assume in each round, possible win = possible loss.
- Assume you can bet any amount, if you have the money.
- Before each round you decide how much you will bet, based only on the outcomes of the previous rounds.
- Simple examples:
  - Straight Play: bet 1$ every round.
  - Chicken Play: bet 1$ every round until the first loss, then stop.
- In probability theory, the random variable

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- Algorithm:
  1. Bet 1$.
  2. If you lost, bet 2$.
  3. If lost again, bet 4$.
  4. And so forth, doubling the stake each round until you win.
  5. If you win, go to stage 1.
- Analysis:
  - Suppose you lost 3 times in a row and then won. Your total win (loss) is $-1-2-4+8$
  - In general, when a losing streak is followed by a win, the win pays for the losses and brings you 1$ on top.
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- An excerpt from Casanova's diary, Venice, 1754:

  Playing the martingale, and doubling my stakes continuously, I won every day during the remainder of the carnival. [..] I congratulated myself upon having increased the treasure of my dear mistress [..]

- Alas, several days later,

  I still played on the martingale, but with such bad luck that I was soon left without a sequin. As I shared my property with my mistress, [..] at her request I sold all her diamonds, losing what I got for them; she had now only five hundred sequins by her. There was no more talk of her escaping from the convent, for we had nothing to live on!

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You Cannot Reverse the Odds

- Problem: you need **a lot** of money if a losing streak is long.
- The martingale is a sure thing only with infinite capital!

Theorem

If the game is fair, your average capital remains constant regardless of the betting system.

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If the game is subfair, your average capital **decreases after every round** *regardless of the betting system.*

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Example: Gambler's Ruin with the Martingale

- Are betting systems completely worthless then?
- Remember example: you have 900$, you want to reach 1000$.
- Suppose the game is fair
  - Using Straight Play, the probability to reach the target is 0.9.
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- Suppose you play roulette
  - Using Straight Play, the probability is 0.00003.
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- If the game is subfair, every round we lose some (on average).
- To minimize losses — minimize rounds!
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  1. Suppose you are within x dollar of your target.
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Bold Play is Optimal

Theorem

For every fixed subfair game, starting capital n and the target m, Bold Play provides the best chances of success.

Conclusions

- You cannot reverse the odds.
- If you must play and have a target,
  Bold Play will maximize your chances of reaching it.
- How not to lose your money gambling?

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