Sheet 6

Create the directory sheet06 and make it your working directory.

Example 1. This example is warm-up to learn to use Matlab's DFT algorithm and generate plots.

a) Plot the absolute value of the DFT $\widehat{y} = \{\widehat{y}_k\}$ of $f(x) = x + x^2$, $x \in [0, 2\pi]$ as follows:

```
n = 2∧5;
x = linspace(0,2*pi,n);
w = 1:length(x);
y = x + x.∧2;
yt = fft(y);
plot(w,abs(yt),'o')
xlabel('k');ylabel('Modulus of DFT')
```

Try using $n = 2^6$ and 2^7.

Repeat this for :

b) $f(x) = x\sin(10x)e^{-x}$, $x \in [0, 2\pi]$

c) $f(x) = \cos(10x)\sin(10x)e^{-x}$, $x \in [0, 2\pi]$

d) $f(x) = \sin(30x)\cos(10x)$, $x \in [0, 2\pi]$

e) $f(x) = \begin{cases} -1 & \text{if} \quad 0 \le x \le \pi \\ 1 & \text{if} \quad \pi < x \le 2\pi \end{cases}$

Example 2. In this example, you are going to plot the modulus of the Discrete Fourier Transform of $f(x) = (x-\pi)^2$, $x \in [0, 2\pi]$ along with the n times the modulus of the complex Fourier coefficients to see how they relate to one another.

Proceed as follows. First enter the function and give the desired n

```
clear
f = @ (x) (x-pi).∧2;
n = 2∧3;
```

Find the complex Fourier coefficients β_k:

```
syms x k;
b_k = int(f(x)*exp(-i*k.*x),x,0,2*pi)/(2*pi)
```

Remember to execute these lines, then copy and paste the output from the command window in the following line:

`kth_coef = @ (k)` paste here

Now compute the coefficients:

```
for m = 1:n-1
  b(m) = int(f(x),x,0,2*pi)/(2*pi);
end
b_0 = kth_coef(0);
```

List all the coefficients (the double converts to decimal form)

```
coeffs = double([b_0,b(1:n-1)]);
```

Find the DFT

```
x = linspace(0,2*pi,n);
y = f(x);
yt = fft(y);
```

Finally, plot the coefficients and the DFT on the same graph:

```
w = 1:length(x);
plot(w,abs(yt),'o',w,n*abs(coeffs),'r','linewidth',2)
xlabel('k');
ylabel('Modulus of DFT and n times Complex Fourier Coefficient')
legend('n|DFT|','|Fourier Coefficient|')
```

Sometimes it is useful to zoom in on what is going on near the $x-$axis. Remember you do this with an `axis` statement, for example in this case, `axis([0 n 0 10])` would be suitable.

Try different values of n, say 2^4 and 2^5.

Aside: Recall if β_k is the complex Fourier coefficient and $\widehat{y} = \{\widehat{y}_k\}$ is the DFT, then we have the approximation $\widehat{y}_k \approx n\beta_k$ for k small compared to n. You should see how this is working as you change the value of n.

Example 3. Repeat for the following (I suggest you save each separately: ex3a.m, ex3b.m, etc.). Again, vary n to see how the approximation changes and make appropriate `axis` statements to zoom in on the action near the $x-$axis, if necessary.

a) $f(x) = x + x^2$, $x \in [0, 2\pi]$

b) $f(x) = xe^{-x}\sin(10x)$, $x \in [0, 2\pi]$

Note that when we are plotting $|\beta_k|$ as a function of $k = 1, \ldots, n$, any peak in the plot at some value $k = k_0$ corresponds to a large frequency component for the frequency $\omega = k_0$.

Because of the sine that appears in $f(x)$, wiggles in the plot of f will occur with an approximate frequency of $\omega = 10$. So you should see a large frequency component in the DFT plot at $k = 10$. Do you?

c) $f(x) = \cos(10x)\sin(10.2x)$, $x \in [0, 2\pi]$

In this example there should be two peaks corresponding to $\omega = 0, 20$ in the first half of the plot of the DFT.

d) $f(x) = e^{-3x}\sin(30x)\cos(10x)$, $x \in [0, 2\pi]$

There should be two peaks in the first half of the DFT. What are the corresponding values of the frequency ω?

e)

$$f(x) = \begin{cases} -1 & \text{if} \quad 0 \le x \le \pi \\ 1 & \text{if} \quad \pi < x \le 2\pi \end{cases}$$

Hint: On this one, you will need to modify the method a bit. In the lines defining `b_k` and `b_0`, enter the integral in two pieces corresponding to $[0, \pi]$ and $[\pi, 2\pi]$ and enter the corresponding value of f in place of $f(x)$.

Sheet 6: Further Exercises

f)

$$f(x) = \begin{cases} \pi^2 - x^2 & \text{if} \quad 0 \leq x \leq \pi \\ -\pi^2 + (x-\pi)^2 & \text{if} \quad \pi < x \leq 2\pi \end{cases}$$