Signal and Image Analysis

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1. How to compress huge data files for transmission over data lines with limited bandwidth?
2. How do eliminate noise or errors in transmitted data?

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- How to efficiently send huge amounts of telemetry from an interplanetary satellite back to Earth?

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Each has its own niche in various applications.

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- This expansion is called a *Fourier Series*

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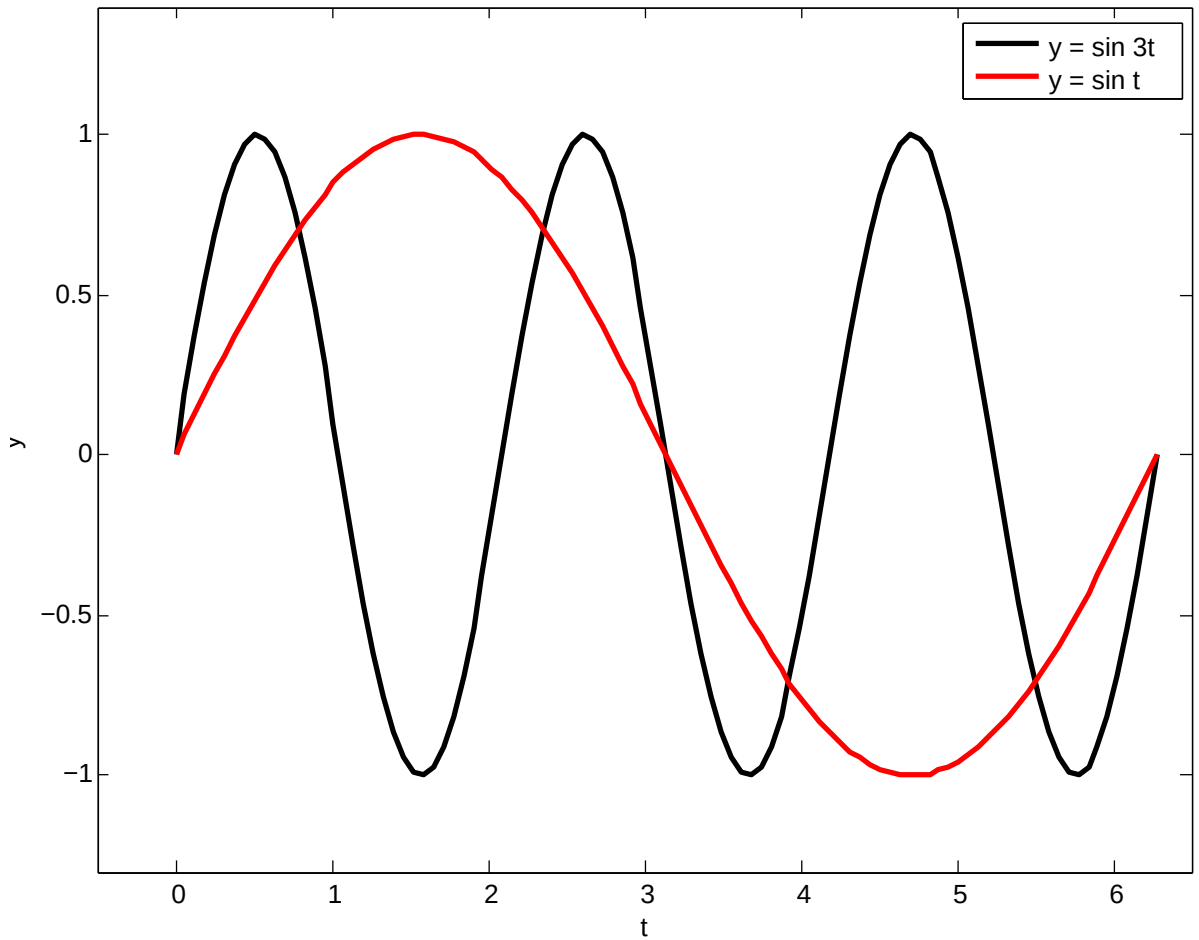
- Thus a high frequency means lots of wiggles:



Applications of Fourier Analysis: Filter Out Noise

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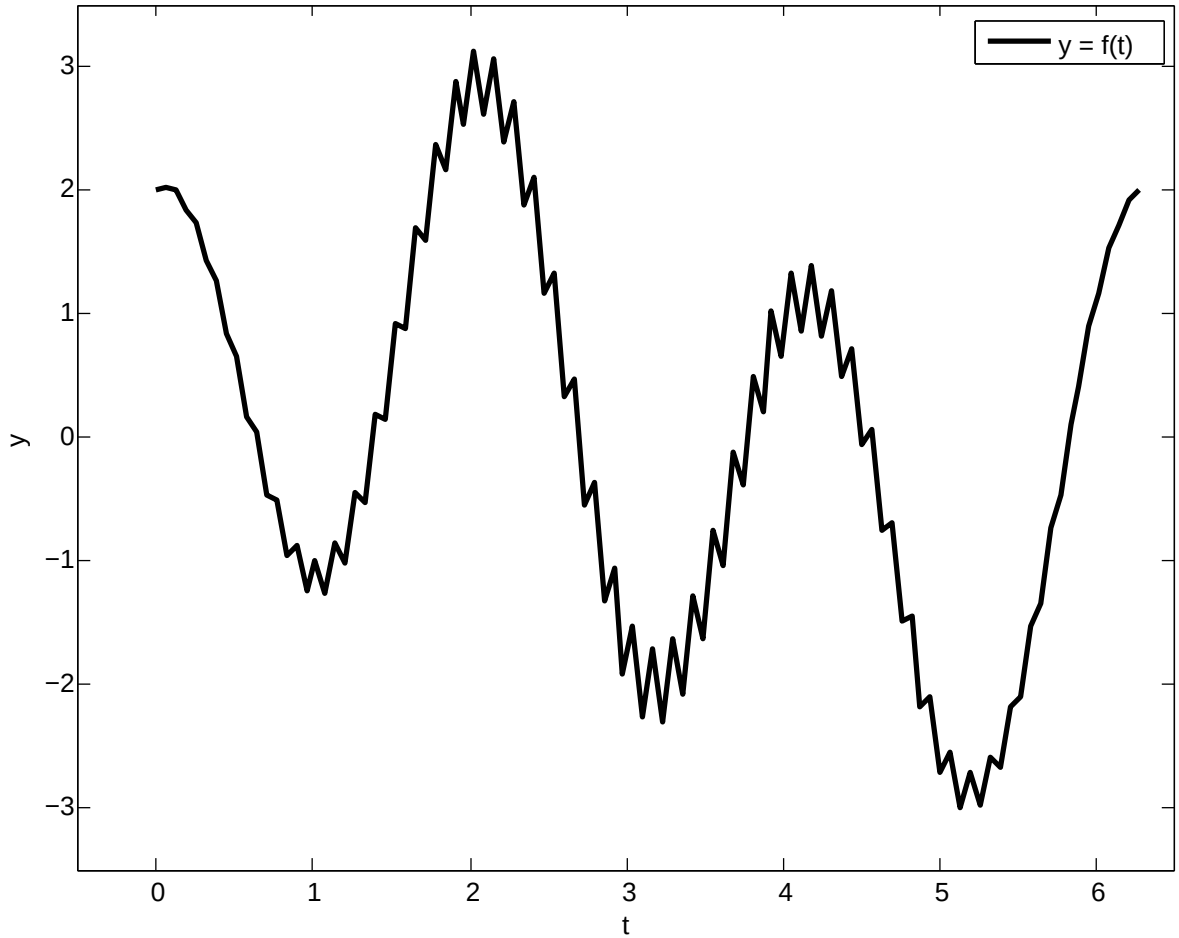

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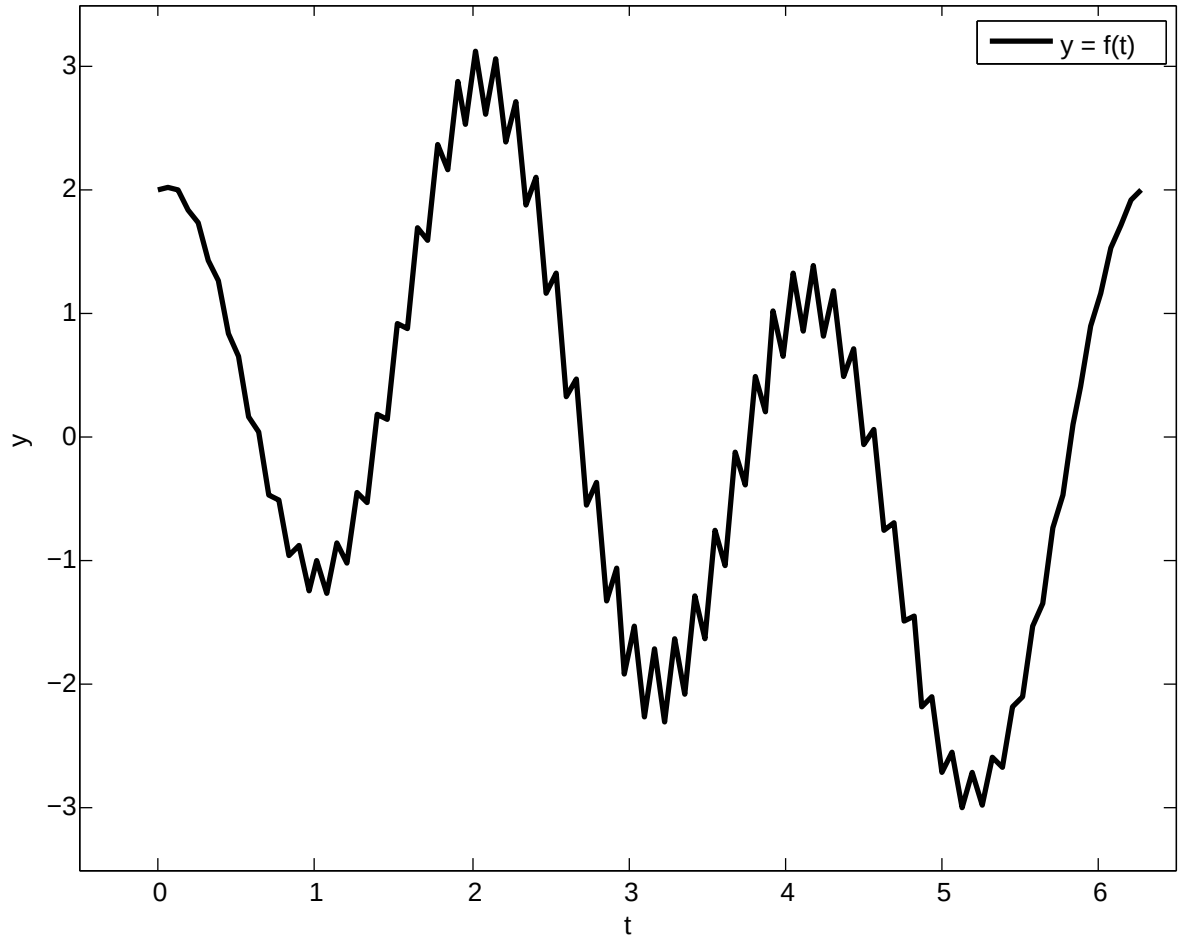

The Fourier expansion of $f(t)$ turns out to be

$$f(t) = \sin(t) + 2\cos(3t) + .3\sin(50t)$$

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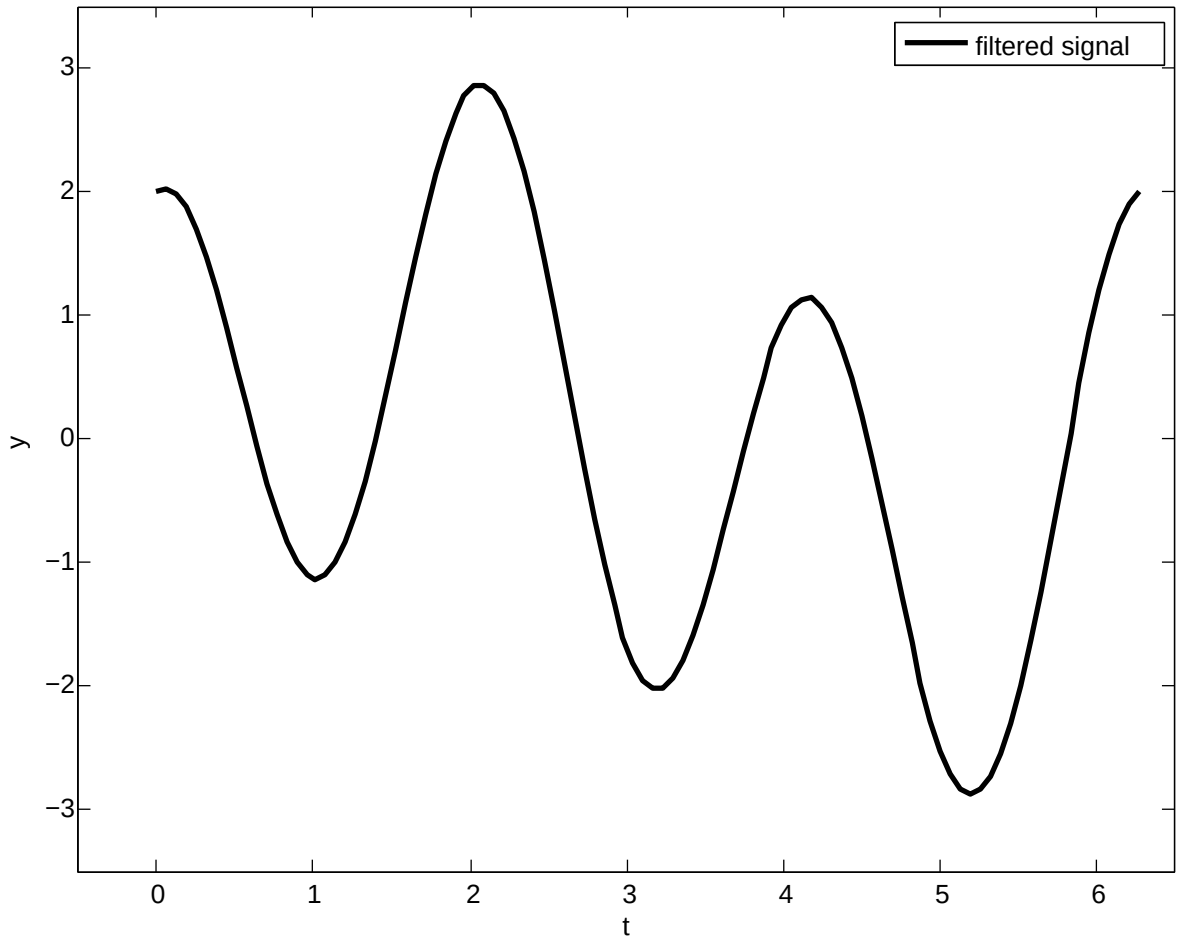
- looks like the noise is due to the high frequency part of $f(t)$
- throw it out:



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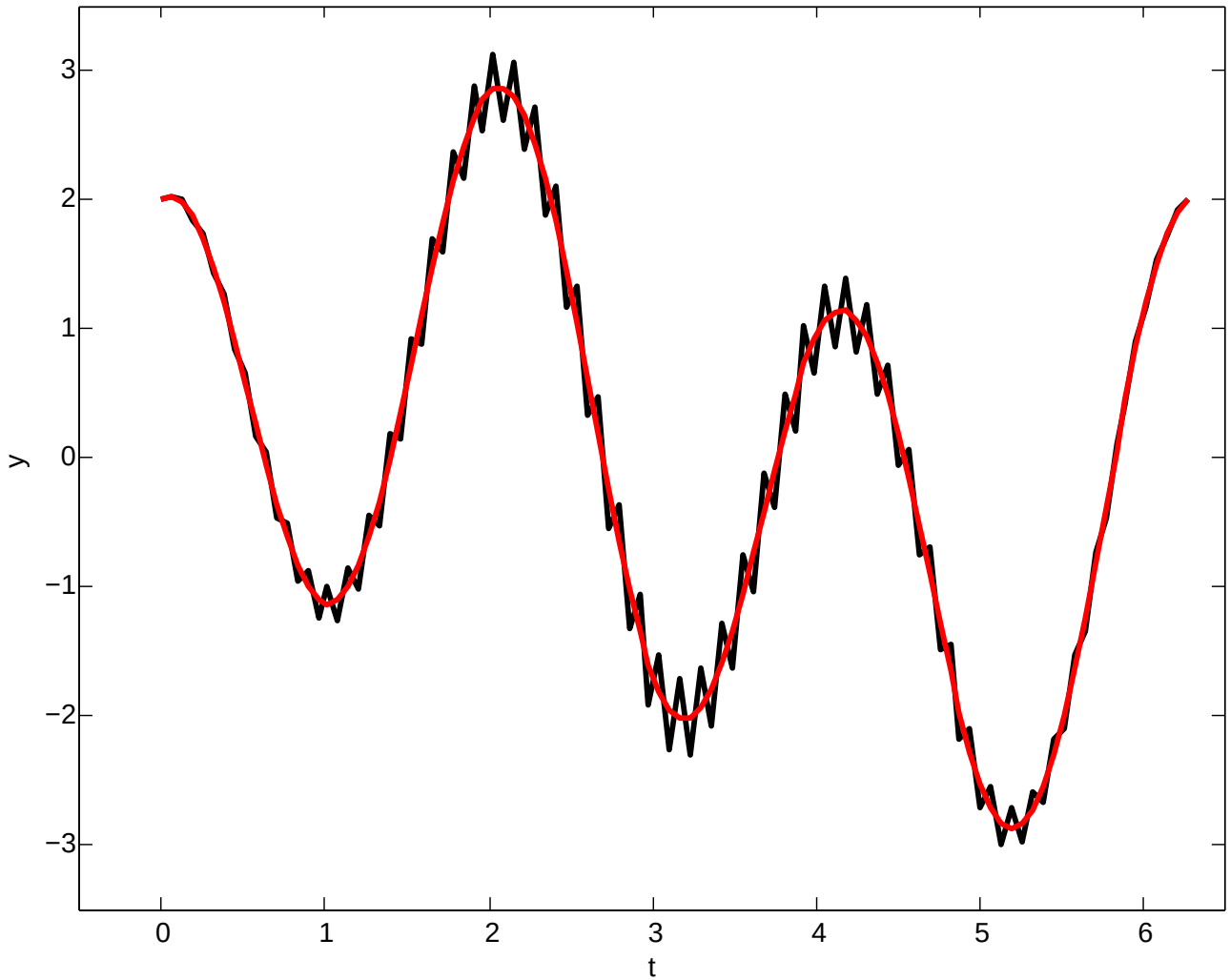

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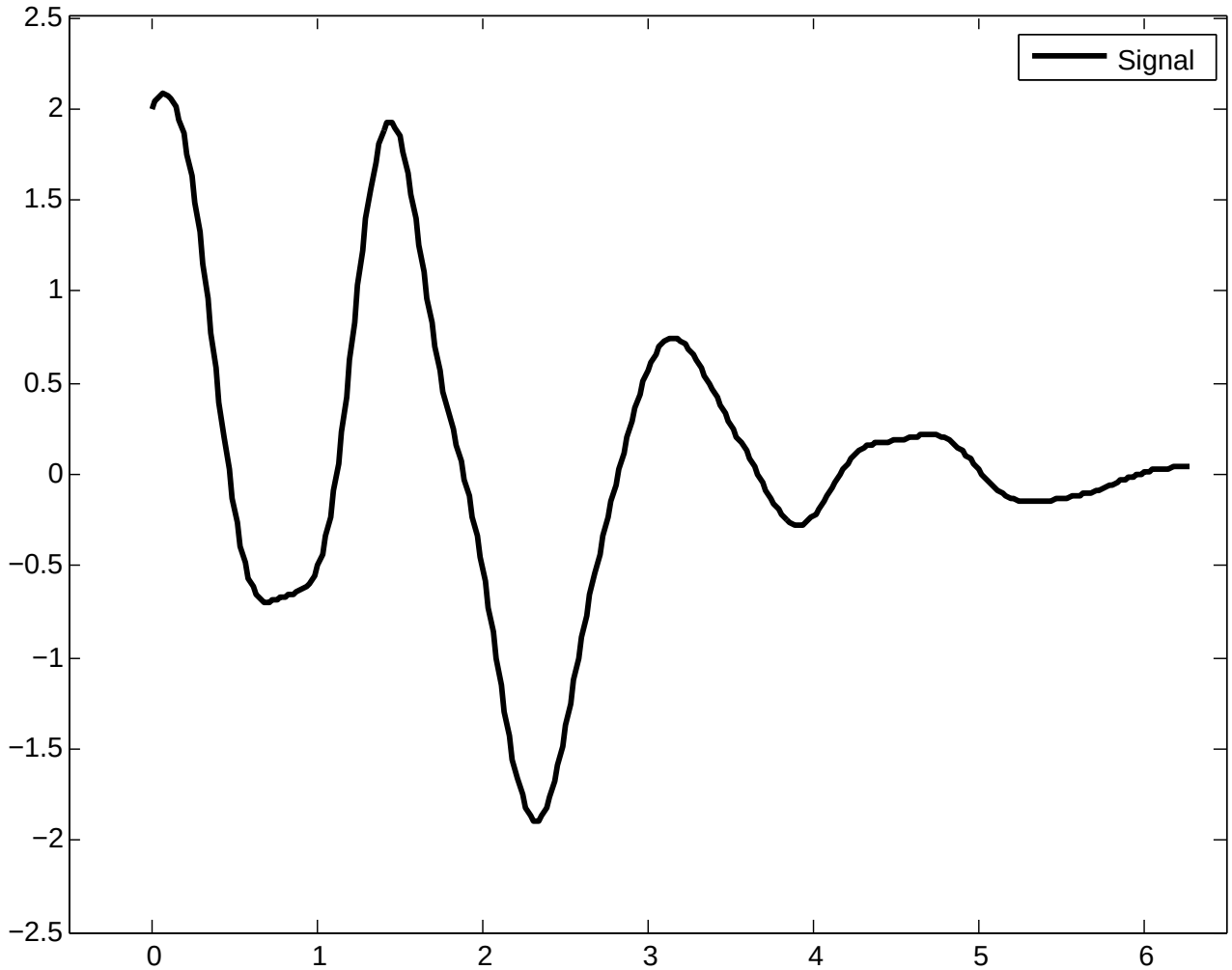
Problem:

- know graph of $f(t)$ only through a set of data points
- how to approximate the Fourier coefficients a_n and b_n from the data?

Applications of Fourier Analysis: Data Compression



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  - this gives thousands of bits of data per second for just one phone call
  - thousands of other calls going on at the same time
  - staggering amount of data

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  - use as few digital bits as possible without distorting the signal too much
  - ideally, the compression is so good that nobody notices the signal has been altered

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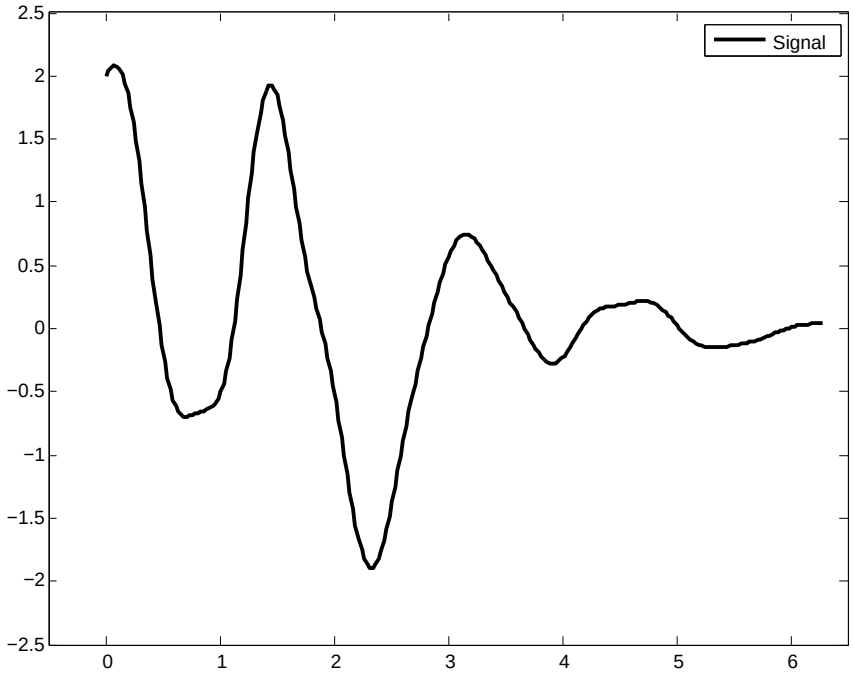
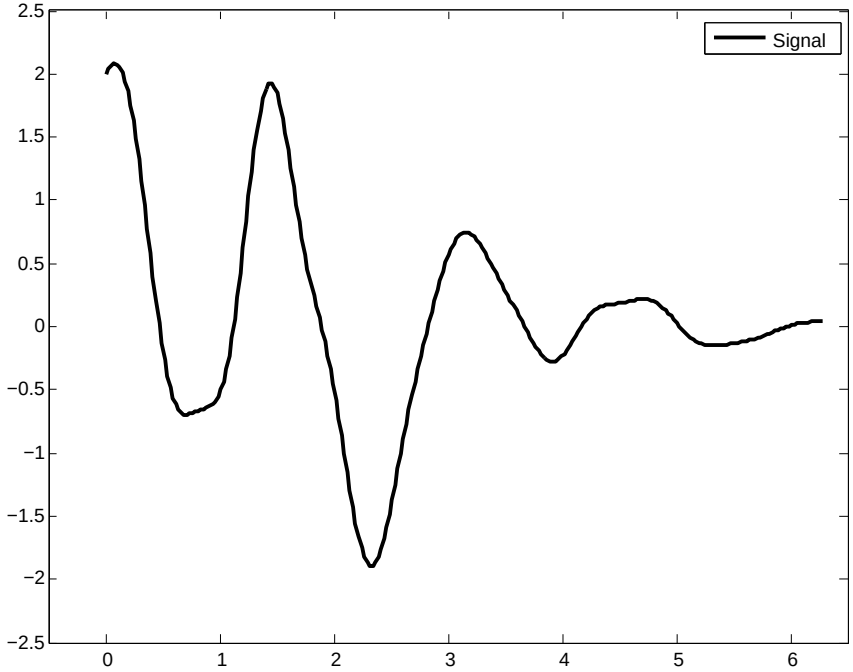
- throw out the coefficients a_n and b_n having absolute value smaller than some preset tolerance
- send only those coefficients that were kept
- for many signals, the number of significant coefficients is relatively small


Signal
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5
0
1
2
3
4
5
6


Signal
2.5
2
1.5
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0
−0.5
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−2.5
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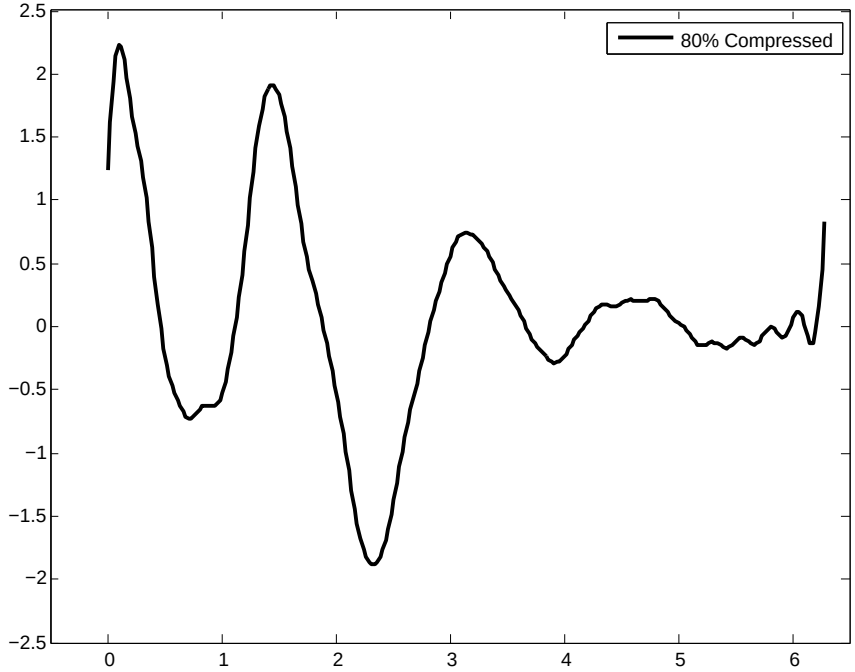
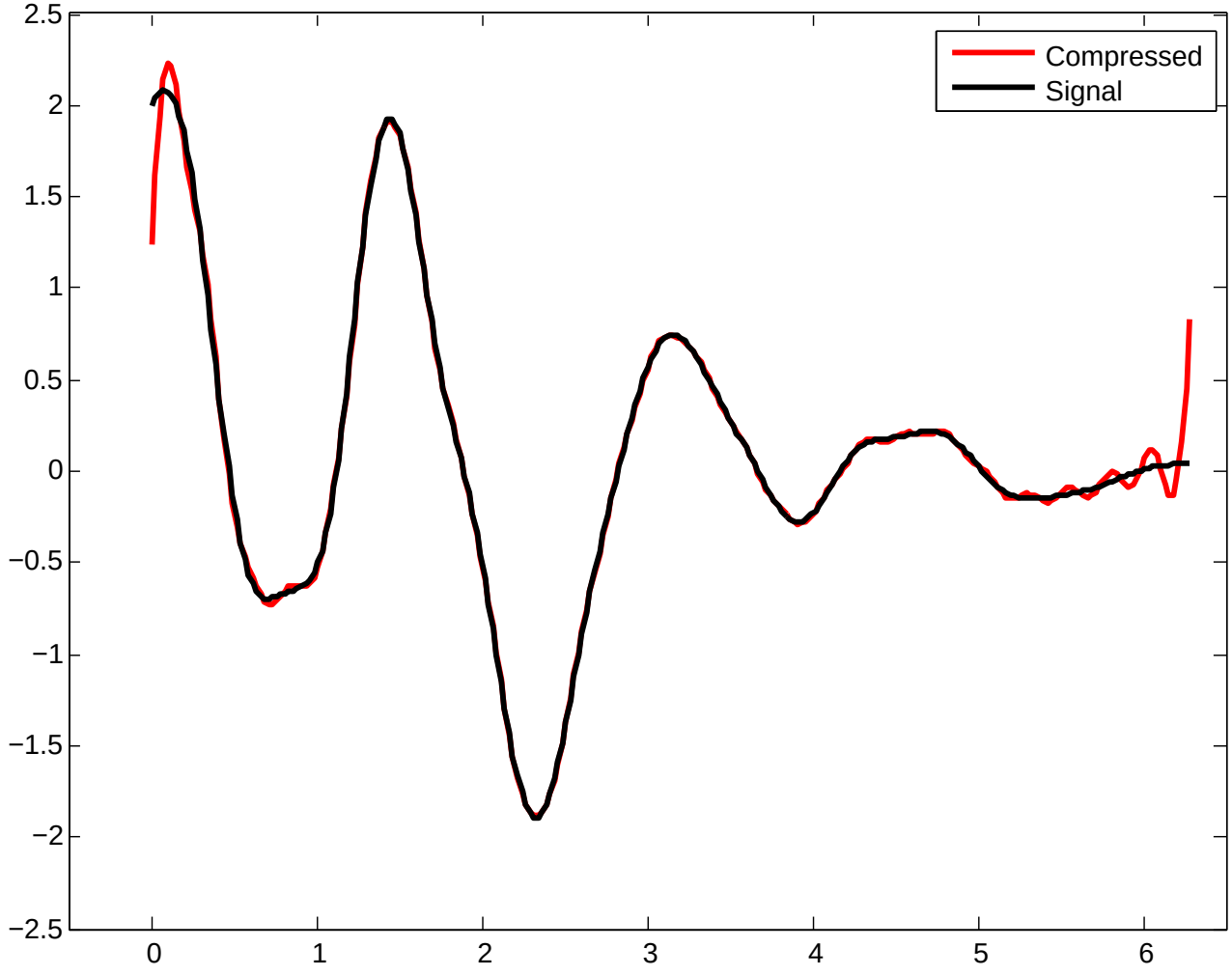
2
3
4
5
6


80% Compressed
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5
0
1
2
3
4
5
6


Compressed
Signal
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5
0
1
2
3
4
5
6

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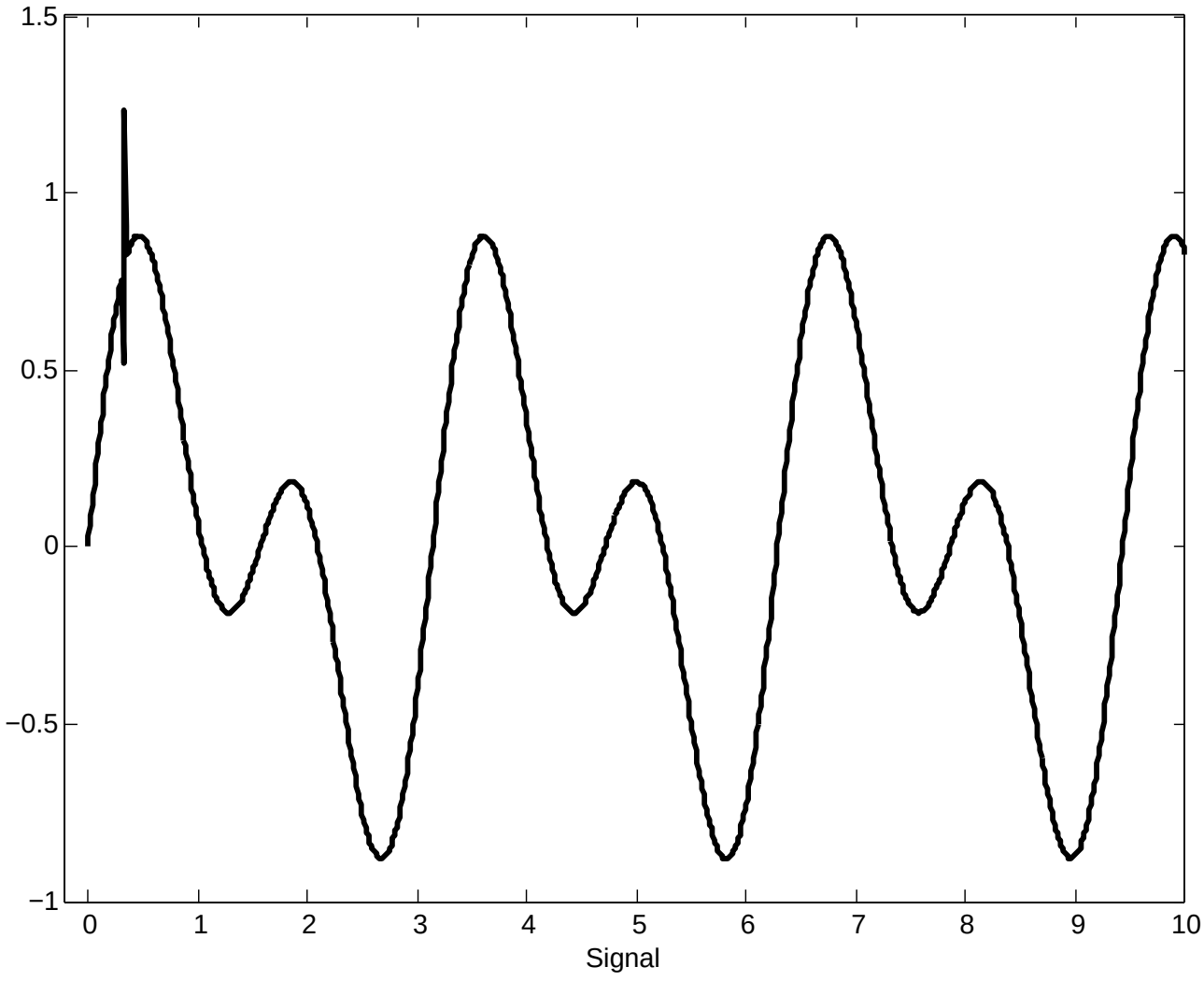
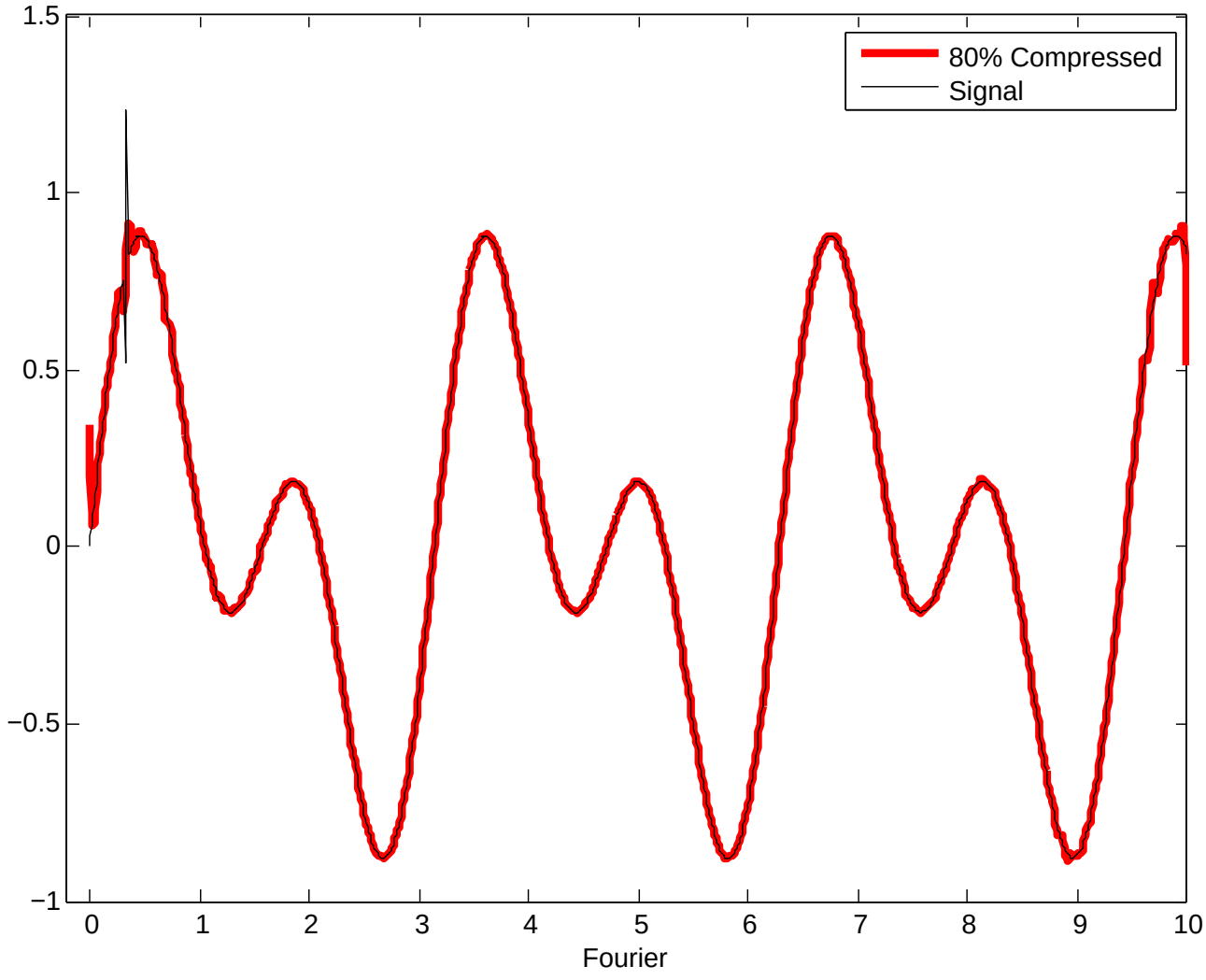
- excellent for signals with time-independent wavelike features with some repetition (for instance, background noise)—no isolated spikes;
- not so good when isolated rapidly occurring spikes or "pops" are present:

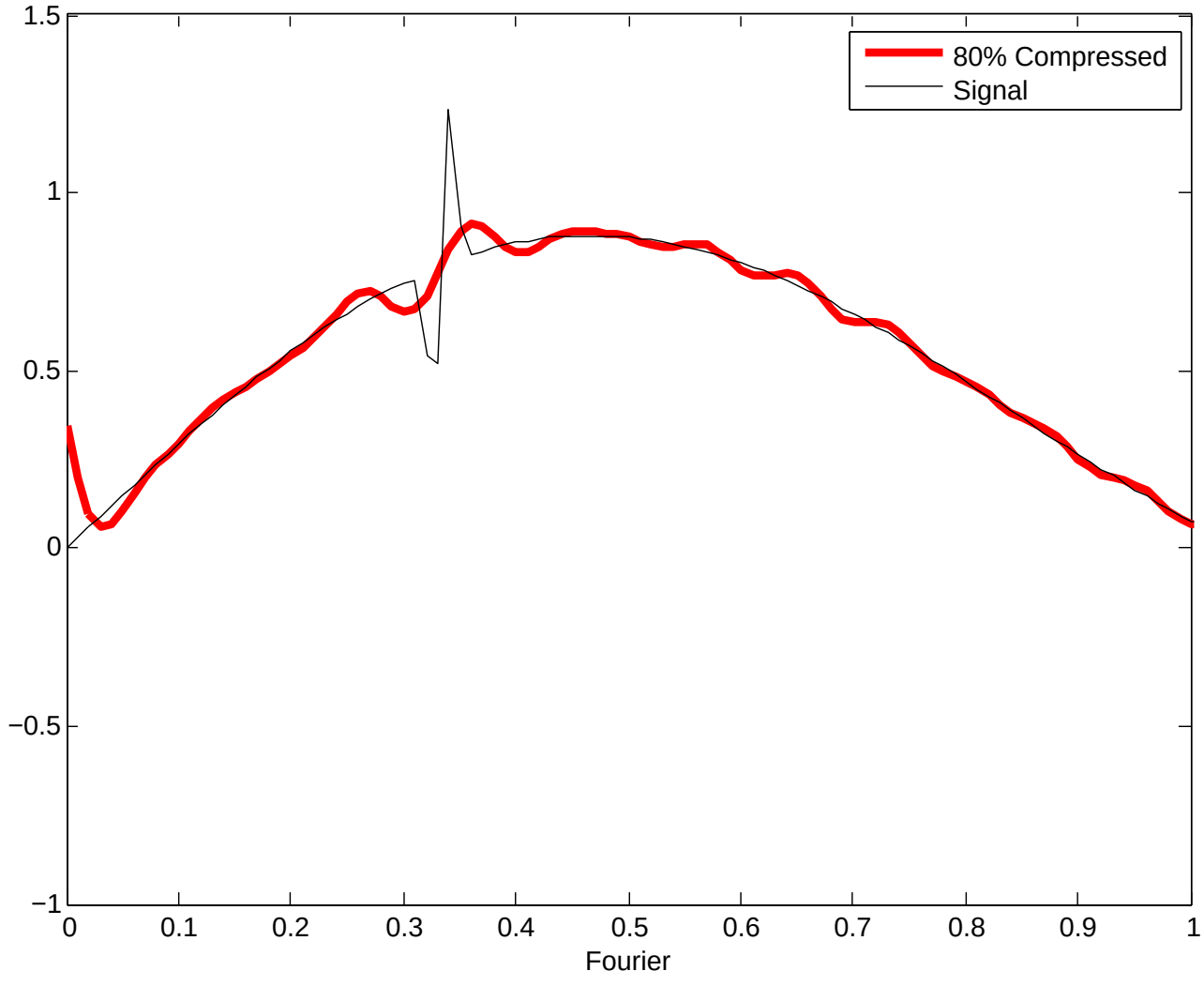


- because of the isolated nature of the spike, Fourier analysis has trouble compressing the signal:



It looks like it missed the spike.

Let's zoom in on the spike to make sure:



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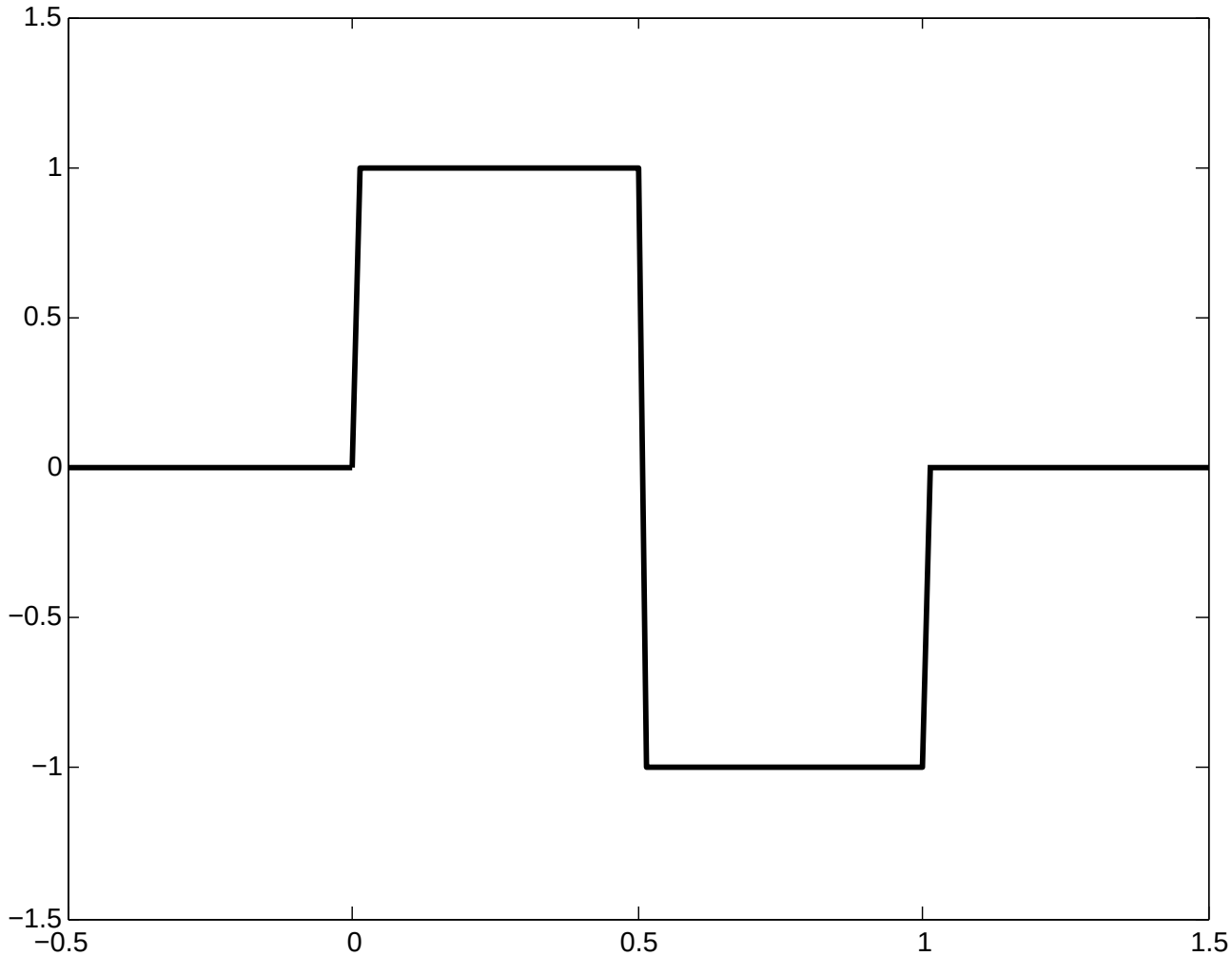
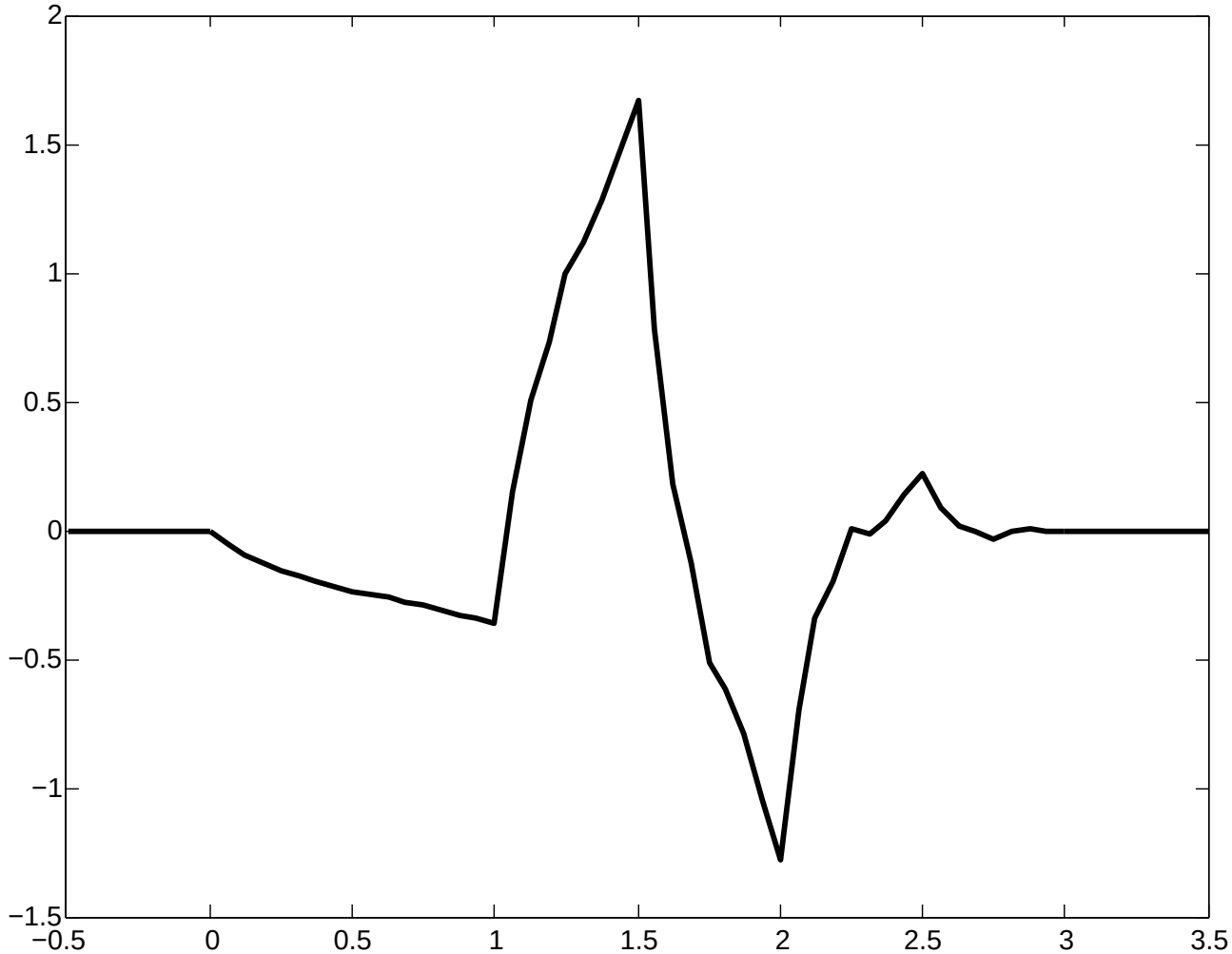
- wave that travels for one or more time periods
- nonzero only over a finite time interval—definitely not periodic!
- complementary tool to Fourier analysis:
  - wavelets are great for signals with isolated spikes

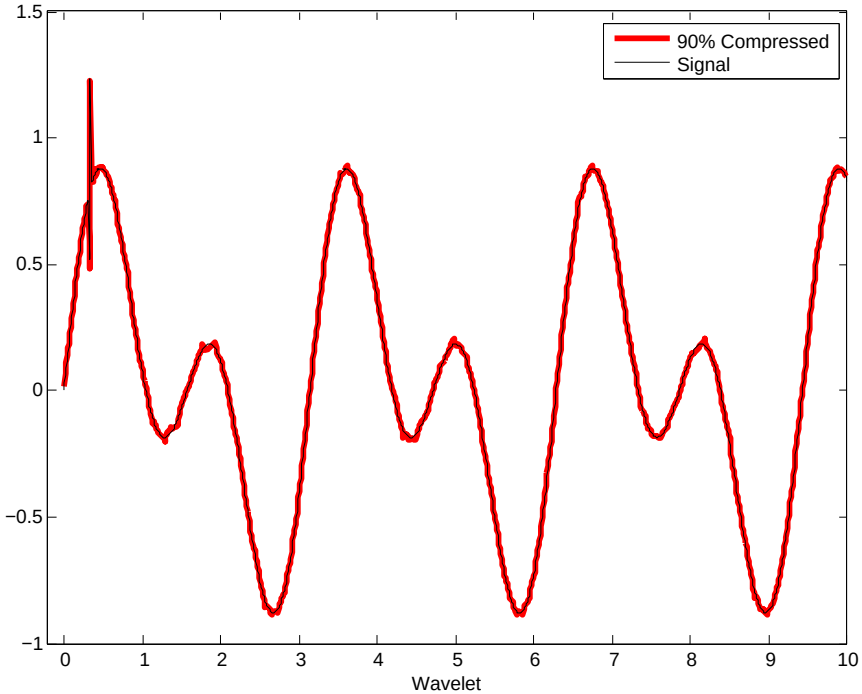
Haar Wavelet



Daubechies Wavelet



Wavelet Compression



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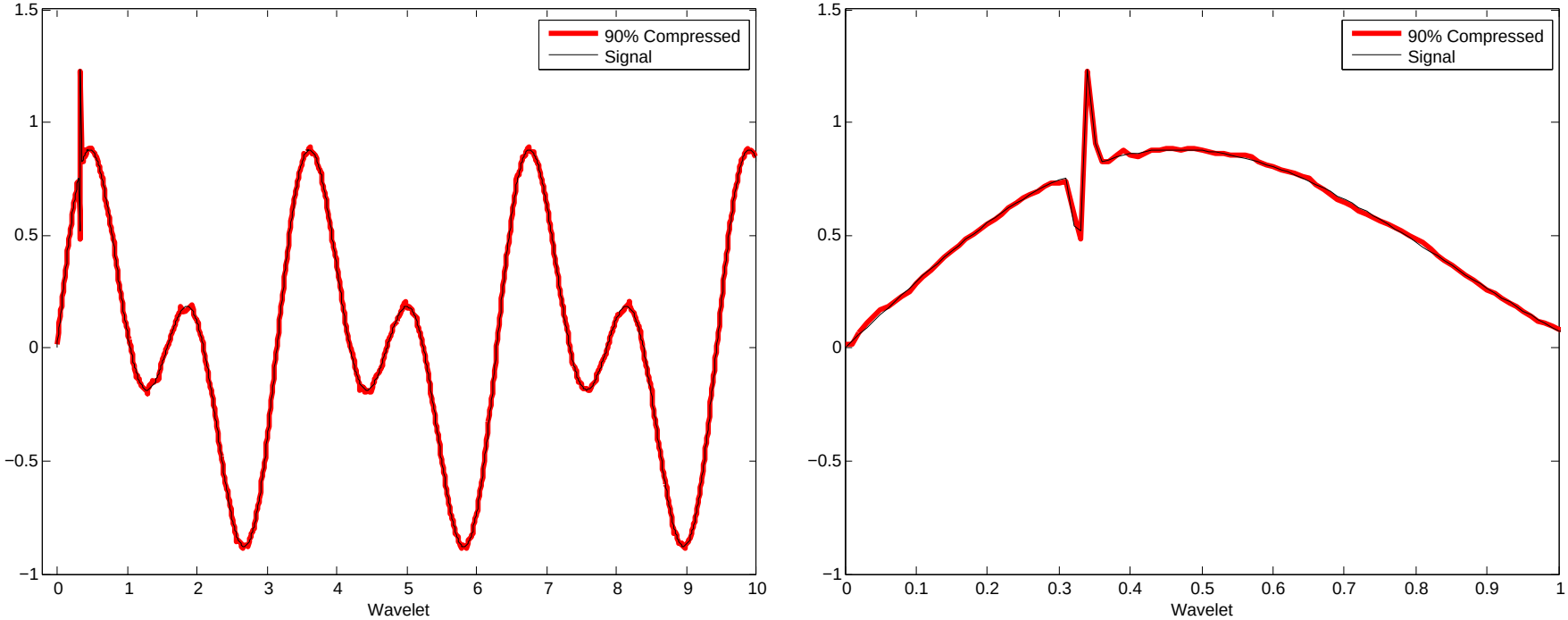

Even at 90% compression, it doesn't miss the spike!