Calculus

Instructions Please write your name in the upper right-hand corner of the page. Write complete sentences to explain your solutions.

1. Suppose $f(x) = 4\cos x$. Show that the Riemann sum for the function f on the interval $[0, \pi/2]$ using left-hand endpoints for the partition $\{0, \pi/6, \pi/4, \pi/3, \pi/2\}$ is equal to $\pi(6 + \sqrt{3} + \sqrt{2}\,)/6$.
   [This is exercise 8 on page 377 of the textbook.]

   Solution. Writing out $\sum_{i=1}^{4} f(x_{i-1})\Delta x_i$ gives the sum

   $$4\cos(0)\left(\frac{\pi}{6} - 0\right) + 4\cos(\pi/6)\left(\frac{\pi}{4} - \frac{\pi}{6}\right) + 4\cos(\pi/4)\left(\frac{\pi}{3} - \frac{\pi}{4}\right) + 4\cos(\pi/3)\left(\frac{\pi}{2} - \frac{\pi}{3}\right),$$

   which simplifies to

   $$4 \cdot \frac{\pi}{6} + 4 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\pi}{12} + 4 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\pi}{12} + 4 \cdot \frac{1}{2} \cdot \frac{\pi}{6}$$

   or $\frac{\pi}{6}(6 + \sqrt{3} + \sqrt{2}\,)$.

2. Express $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} \sin\left(\frac{4i^2}{n^2}\right)$ as a definite integral.

   Solution. The solution is not unique, but perhaps the most natural interpretation of the expression is the limit of Riemann sums for regular partitions of the interval $[0, 2]$ into n subintervals each of width $2/n$, where right-hand endpoints are used, and the function is $\sin(x^2)$. The limit is then equal to the definite integral $\int_0^2 \sin(x^2)\,dx$.

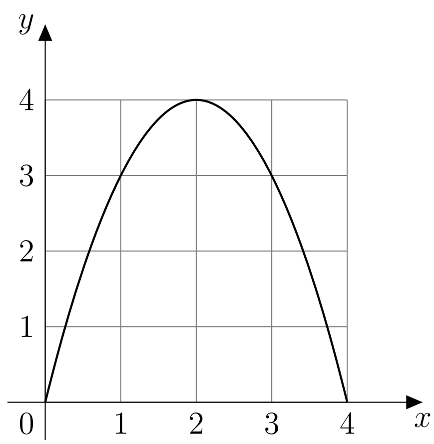
   Another interpretation is that the interval is $[0, 1]$, the n subintervals of the partition each have width $1/n$, the function is $2\sin(4x^2)$, and the limit is equal to the definite integral $\int_0^1 2\sin(4x^2)\,dx$.

Calculus

3. Let $p_1 = 2$, $p_2 = 3$, $p_3 = 5$, and in general let p_n be the nth prime number. What is the smallest value of k for which $\displaystyle\sum_{i=1}^{k} \frac{1}{p_i} > 1$?

   Solution. When $k = 1$, the sum equals $\frac{1}{2}$. When $k = 2$, the sum equals $\frac{1}{2} + \frac{1}{3}$, or $\frac{5}{6}$. When $k = 3$, the sum equals $\frac{1}{2} + \frac{1}{3} + \frac{1}{5}$, or $\frac{31}{30}$. Thus the sum first exceeds 1 when $k = 3$.

4. The figure shows the graph of a function f. Estimate the value of $\int_0^4 f(x)\,dx$ by using a regular partition with four subintervals and choosing right-hand endpoints.



   Solution. The estimate is the shaded area below: namely, 10.