Calculus

Instructions Please write your name in the upper right-hand corner of the page.

If you need more space to write your solutions, you may use the back of the page or a separate sheet of paper.

1. How is the word "asymptote" spelled?

   Solution. The spelling printed above is correct: `asymptote`. In standard dialects of English, the letter "p" is not pronounced in this word.

2. The line that passes through the two points $(1, -1)$ and $(4, 5)$ has the standard Cartesian equation $y = 2x - 3$. Find an equation for this line either in the vector form $\vec{r}(t) = \vec{r}_0 + t\vec{v}$ or in the parametric form $x(t) = x_0 + at$ and $y(t) = y_0 + bt$.

   Solution. The vector $\vec{v}$ must be parallel to the vector joining the two points: namely, $\langle 4 - 1, 5 - (-1)\rangle$ or $\langle 3, 6\rangle$. The point $\vec{r}_0$ can be any point on the line; a natural choice for $\vec{r}_0$ would be either of the two given points. Thus one possible solution is $\vec{r}(t) = \langle 1, -1\rangle + t\langle 3, 6\rangle$, and another possible solution is $\vec{r}(t) = \langle 4, 5\rangle + t\langle 3, 6\rangle$.

   Any vector parallel to $\langle 3, 6\rangle$ will do just as well, and the vector $\langle 1, 2\rangle$ is a natural choice. Therefore either $\vec{r}(t) = \langle 1, -1\rangle + t\langle 1, 2\rangle$ or $\vec{r}(t) = \langle 4, 5\rangle + t\langle 1, 2\rangle$ is correct too. In fact, there are infinitely many correct solutions, since you could take any scalar multiple of $\langle 1, 2\rangle$ for the vector $\vec{v}$, and you could take any point on the line as the base point $\vec{r}_0$. For instance, another correct solution is $\vec{r}(t) = \langle 0, -3\rangle + t\langle 2, 4\rangle$.

   One way to check your solution is to pick two values of t, say $t = 0$ and $t = 1$, and substitute them into your vector equation to obtain two points on the line. If these two points satisfy the Cartesian equation $y = 2x - 3$, then your vector equation is correct.

   You could also write the vector equation using the standard unit basis vectors $\vec{i}$ and $\vec{j}$. Thus instead of writing $\vec{r}(t) = \langle 1, -1\rangle + t\langle 1, 2\rangle$, you could write $\vec{r}(t) = (1 + t)\vec{i} + (-1 + 2t)\vec{j}$. That form of the answer translates directly into the equivalent parametric equations $x(t) = 1+t$ and $y(t) = -1 + 2t$.

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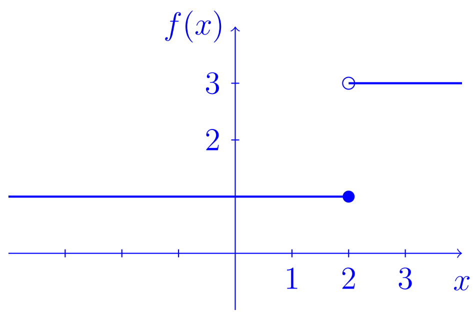
There are infinitely many correct solutions in parametric form. Again, you can check your solution by picking two values of t and verifying that you get two points on the line with Cartesian equation $y = 2x - 3$.

3. Draw a graph that illustrates a function f with the properties that $\lim_{x\to 2^-} f(x) = 1$ and $\lim_{x\to 2^+} f(x) = 3$.

   Solution. There are many different correct solutions. There is a jump in the graph above $x = 2$; the height of the graph approaches 1 as x approaches the value 2 from the left, and the height of the graph approaches 3 as x approaches the value 2 from the right. One such graph is shown below.



   The open circle and the filled-in circle indicate in this picture that $f(2) = 1$, but the value of f precisely at $x = 2$ has no effect on either of the limits.

4. According to the precise definition of a limit, the meaning of the symbols "$\lim\limits_{x\to a} f(x) = L$" is that for every positive number ε there corresponds a positive number δ such that [fill in the blanks]

   ______________________ whenever ______________________ .

   Solution. $|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$.