

- Consider the statement form $P \Rightarrow (Q \vee R)$. Write in symbols
 - the converse of the statement;
 - the contrapositive of the statement;
 - a logically equivalent statement that does not use the symbol " \Rightarrow ".
- In this problem the universe U is the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Suppose $A = \{2, 4, 6, 8\}$, and $B = \{1, 3, 5, 7, 9\}$. Match each set in the first column with a set that it equals in the second column.

$A \cup B$		A
$A \cap B$		B
$B - A$		\emptyset
$A \cup \overline{B}$		U

- Write in words the negation of each of the following statements without using the word "not".
 - Every mathematics book is expensive.
 - Either I will do well on this test or I will be sad. [inclusive "or"]
 - If I had a good night's sleep, then I am thinking clearly.
- For each of the following statement forms about sets A , B , and C , say whether it is true or false.
 - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
 - $\overline{A \cap B} = \overline{A} \cap \overline{B}$
- Prove by induction that

$$1 + 2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 1$$

for every positive integer n .

- Define the symbol " \odot " by the rule that the statement form $P \odot Q$ is false when P and Q are both true, and $P \odot Q$ is true otherwise. (See the truth table in the margin.)

P	Q	$P \odot Q$
T	T	F
T	F	T
F	T	T
F	F	T

Show that the operation \odot is not associative. In other words, show that the statement forms $(P \odot Q) \odot R$ and $P \odot (Q \odot R)$ are not logically equivalent.

Extra credit

The mathematician Charles Lutwidge Dodgson (1832–1898), better known by his pen name, Lewis Carroll, helped popularize a type of logic puzzle called *sorites* (from the Greek word for “heap”). Here is an example from his book *Symbolic Logic*. What logical deduction can be made by using *all ten* of the following statements?

- (i) The only animals in this house are cats.
- (ii) Every animal is suitable for a pet, that loves to gaze at the moon.
- (iii) When I detest an animal, I avoid it.
- (iv) No animals are carnivorous, unless they prowl at night.
- (v) No cat fails to kill mice.
- (vi) No animals ever take to me, except what are in this house.
- (vii) Kangaroos are not suitable for pets.
- (viii) None but carnivora kill mice.
- (ix) I detest animals that do not take to me.
- (x) Animals, that prowl at night, always love to gaze at the moon.