Sets

Vocabulary:

- element
- subset
- complement
- empty set
- cardinality
- interval

Some examples of sets

- **Z** or $\mathbb{Z}$, the integers (0, ± 1, ± 2, $\ldots$)
- $\mathbf{Z}^+$ or $\mathbb{Z}^+$ or $\mathbb{N}$, the positive integers (natural numbers)
- **Q** or $\mathbb{Q}$, the rational numbers, like $2/3$ and $17/220$
- **R** or $\mathbb{R}$, the real numbers (includes rational numbers and also some other numbers like π, e, $\phi = \frac{1+\sqrt{5}}{2}$)
- **C** or $\mathbb{C}$, the complex numbers, like $2 + 3i$ where $i^2 = -1$
- $\varnothing$, the empty set, not to be confused with Greek letter ϕ

Elements

$a \in A$ means "a is an element of A."
Do not confuse set membership $\in$ with the Greek letter ϵ.
$a \notin A$ means "a is not an element of A."

Set-builder notation:

- $\{\, x \in \mathbf{R} \mid x^2 - 1 < 0 \,\}$ or $\{\, x \in \mathbb{R} : x^2 - 1 < 0 \,\}$ is the set of real numbers between -1 and 1, the *open interval* $(-1, 1)$.
- $[-1, 1]$ is the *closed interval* of real numbers between -1 and 1 inclusive.

More intervals

- $[-1, 1)$ half-open, half-closed interval
- (a, ∞) means the *unbounded* open interval $\{\, x \in \mathbf{R} \mid a < x \,\}$.

Subsets

$A \subseteq B$, "A is a subset of B," means that every element of A is also an element of B.
$A \subset B$, "A is a proper subset of B," means that A is a subset of B but $A \neq B$. Sometimes written $A \subsetneq B$.
Some authors do not distinguish between the symbols $\subset$ and $\subseteq$.
So $\mathbf{Z} \subset \mathbf{Q} \subset \mathbf{R} \subset \mathbf{C}$.
Question: is $\varnothing \subseteq \mathbf{Z}$? Yes, because there is no element in the empty set to contradict the statement.

Set complements

$B - A$ or $B \setminus A$ means $\{\, x \in B \mid x \notin A \,\}$, "the complement of A in B."

$\mathbf{Z} \setminus \mathbf{Z}^+$ is the set of nonpositive integers.

$\mathbf{Q} - \mathbf{R} = \varnothing$

When B is an implicit universal set, the complement of A can be written $\overline{A}$ or A' or A^c.

Cardinality

When A is a finite set, the number of elements of A is the *cardinality* of A, written $|A|$.

Equality of sets

When A and B are sets, to say that $A = B$ is equivalent to the conjunction

$$(A \subseteq B) \wedge (B \subseteq A).$$