Vocabulary

- function
- domain
- codomain
- image
- inverse image
- graph

Functions

A *function* (also called a *mapping*) consists of three things:

1. a set A, called the *domain*,
2. a set B, called the *codomain*,
3. an assignment of each element of A to some element of B.

Standard notation for a function f from domain A to codomain B is $f : A \to B$.

Examples

- $A = \mathbf{Z}$, $B = \mathbf{R}$, $f(n) = \sqrt{n^2 + 1}$
- $A =$ set of Aggies, $B = \mathbf{Z}$, $f(a) =$ the Aggie's age in years.
- $A =$ set of Aggies, $B = \mathbf{C}$, $f(a) =$ the Aggie's age in years.

The last two examples are different functions because they have different codomains.

Image

The *image* (or range) of a function $f : A \to B$ means $\{ b \in B \mid \exists a \in A \text{ such that } f(a) = b \}$.
Example: $A =$ people in this room, $B = \mathbf{Z}$, $f(a) =$ number of the birth month.
Image is $\{1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12\}$

Inverse image

Suppose $f\colon A \to B$ is a function with domain A and codomain B.
If C is a subset of the codomain, then the inverse image of C is the set $\{a \in A \mid f(a) \in C\}$.
In the birthday month example, the inverse image of $\{6, 7\}$ is $\{$Dr. Boas$\}$.
Example. $A = \mathbf{R}$, $B = \mathbf{R}$, $f(x) = x^2$, $C =$ even integers. The inverse image of C is $\{0, \pm\sqrt{2}, \pm 2, \pm\sqrt{6}, \dots\}$.

Notation

If $f\colon A \to B$ is a function, and J is a subset of A, then $f(J)$ means $\{f(j) \mid j \in J\}$, the *image* of J.
If R is a subset of B, then the inverse image is $f^{-1}(R)$, that is, $\{a \in A \mid f(a) \in R\}$.