Linear Algebra

Write your **name**: ________________________________ (2 points).

In **problems 1–5**, circle the correct answer. (5 points per problem)

1. If the product AB of two matrices A and B is the zero matrix, then either A or B (or both) must be the zero matrix. `True` `False`

2. If A is a 4×5 matrix and $\mathbf{b}$ is a 4×1 matrix (that is, a column vector), then the linear system $A\mathbf{x} = \mathbf{b}$ must have infinitely many solutions for $\mathbf{x}$. `True` `False`

3. If A and B are 3×3 matrices such that $\det(A) = \det(B)$, then the matrices A and B must be the same. `True` `False`

4. If A is a singular 3×3 matrix, then the homogeneous linear system $A\mathbf{x} = \mathbf{0}$ must have nontrivial solutions. `True` `False`

5. If n vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are a spanning set for the vector space R^n, then they must also be a linearly independent set. `True` `False`

In **problems 6–9**, fill in the blanks. (7 points per problem)

6. $\det \begin{pmatrix} \boxed{} & 1 & 2 \\ 0 & \boxed{} & 3 \\ 0 & 0 & \boxed{} \end{pmatrix} = 10.$ (There are many correct answers.)

7. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ and $AB = \begin{pmatrix} 10 & 2 & 3 \\ 40 & 5 & 6 \end{pmatrix}$, then $B = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$.

8. Let V be a vector space. The number of vectors in a basis for V is called the ____________________ of the vector space V.

9. Suppose $A = \begin{pmatrix} 1 & 5 \\ 2 & 11 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix}$. (This is an LU factorization for A.) The transpose matrix A^T can be written as the product of a lower triangular matrix times an upper triangular matrix as follows: $A^T = \begin{pmatrix} 1 & 0 \\ \boxed{} & 1 \end{pmatrix} \begin{pmatrix} 1 & \boxed{} \\ 0 & 1 \end{pmatrix}$.

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In **problems 10–12**, show your work and explain your method. Continue on the back if you need more space. (15 points per problem)

10. Consider the system of simultaneous equations

$$\begin{cases} x_1 + x_2 + 2x_3 = 1 \\ 2x_1 + x_2 + 3x_3 = 2 \\ 4x_1 + 2x_2 + 6x_3 = k \end{cases}$$

for the unknowns x_1, x_2, and x_3. For which value(s) of k does this system have infinitely many solutions?

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11. Find a basis for the null space of the matrix $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 2 & 3 & 4 \\ 1 & -1 & 0 & -2 \end{pmatrix}$.

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12. Suppose $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ \square & \square & \square \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} 4 & 0 & \square \\ \square & \square & \square \\ \square & \square & 1 \end{pmatrix}$.

Fill in the missing entries, and explain your strategy for finding them.