Examination 1 Linear Algebra

Write your **name**:

(2 points).

In **problems 1–5**, circle the correct answer. (5 points per problem)

- 1. If the product AB of two matrices A and B is the zero matrix, then either A or B (or both) must be the zero matrix. True False
- 2. If A is a 4×5 matrix and **b** is a 4×1 matrix (that is, a column vector), then the linear system $A\mathbf{x} = \mathbf{b}$ must have infinitely many solutions for **x**. True False
- 3. If A and B are 3×3 matrices such that det(A) = det(B), then the matrices A and B must be the same. True False
- 4. If A is a singular 3×3 matrix, then the homogeneous linear system $A\mathbf{x} = \mathbf{0}$ must have nontrivial solutions. True False
- 5. If *n* vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are a spanning set for the vector space \mathbb{R}^n , then they must also be a linearly independent set. True False

In **problems 6–9**, fill in the blanks. (7 points per problem)

6. det
$$\begin{pmatrix} 1 & 2 \\ 0 & 3 \\ 0 & 0 \end{pmatrix} = 10.$$
 (There are many correct answers.)

7. If
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$
 and $AB = \begin{pmatrix} 10 & 2 & 3 \\ 40 & 5 & 6 \end{pmatrix}$, then $B = \begin{pmatrix} \\ \\ \end{pmatrix}$.

- 8. Let V be a vector space. The number of vectors in a basis for V is called the ______ of the vector space V.
- 9. Suppose $A = \begin{pmatrix} 1 & 5 \\ 2 & 11 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix}$. (This is an *LU* factorization for *A*.) The transpose matrix A^T can be written as the product of a lower triangular matrix times an upper triangular matrix as follows: $A^T = \begin{pmatrix} 1 & 0 \\ \Box & 1 \end{pmatrix} \begin{pmatrix} 1 & \Box \\ 0 & 1 \end{pmatrix}$.

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In **problems 10–12**, show your work and explain your method. Continue on the back if you need more space. (15 points per problem)

10. Consider the system of simultaneous equations

$$\begin{cases} x_1 + x_2 + 2x_3 = 1\\ 2x_1 + x_2 + 3x_3 = 2\\ 4x_1 + 2x_2 + 6x_3 = k \end{cases}$$

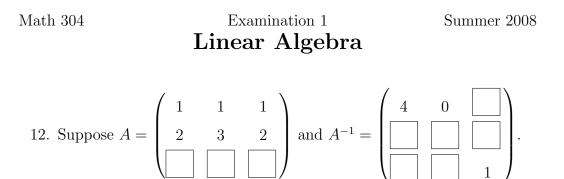
for the unknowns x_1 , x_2 , and x_3 . For which value(s) of k does this system have infinitely many solutions?

Math 304

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Summer 2008

11. Find a basis for the null space of the matrix	/1	0	1	0
11. Find a basis for the null space of the matrix	1	2	3	4.
	$\backslash 1$	-1	0	-2/



Fill in the missing entries, and explain your strategy for finding them.