Linear Algebra

Instructions   Please write your name in the upper right-hand corner of the page. Circle the correct answer in each of the following items.

1. The set of vectors $\left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$ forms an orthonormal basis for R^2.

   True   False

2. In the space $C[-\pi, \pi]$ of continuous functions on the interval $[-\pi, \pi]$ equipped with the inner product

$$\langle f(x), g(x) \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)g(x)\, dx,$$

   the functions $\sin(x)$ and $\cos(x)$ are orthogonal.   True   False

3. In a vector space that is equipped with a norm, the distance between vectors $\mathbf{x}$ and $\mathbf{y}$ is defined to be $\|\mathbf{y} - \mathbf{x}\|$.   True   False

4. Every orthogonal matrix is invertible.   True   False

5. For every pair of vectors $\mathbf{u}$ and $\mathbf{v}$ in an inner product space, the following inequality holds: $|\langle \mathbf{u}, \mathbf{v} \rangle| \leq \|\mathbf{u}\|\, \|\mathbf{v}\|$.   True   False