Linear Algebra

Instructions Please write your name in the upper right-hand corner of the page. Use complete sentences, along with any necessary supporting calculations, to answer the following questions.

1. Find a basis for the column space of the matrix $\begin{pmatrix} 1 & 1 & 1 & 5 \\ 2 & 2 & 1 & 8 \\ 3 & 3 & 2 & 13 \end{pmatrix}$.

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2. In the space $C[0,1]$ of continuous functions on the interval $[0,1]$, the functions e^x and e^{-x} span a two-dimensional subspace. One basis for this subspace, call it the E basis, is $[e^x, e^{-x}]$. Another basis, call it the H basis, is $[\cosh(x), \sinh(x)]$, where the so-called hyperbolic functions are defined as follows:

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \text{and} \qquad \sinh(x) = \frac{e^x - e^{-x}}{2}.$$

Find the transition matrix A from the H basis to the E basis. In other words, find the 2×2 matrix A with the property that if

$$\begin{aligned} f(x) &= a\cosh(x) + b\sinh(x) \\ &= ce^x + de^{-x}, \end{aligned}$$

then $A \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix}$.