

# Linear Algebra

**Instructions** Please write your name in the upper right-hand corner of the page. Use complete sentences, along with any necessary supporting calculations, to answer the following questions.

1. Find a basis for the column space of the matrix  $\begin{pmatrix} 1 & 1 & 1 & 5 \\ 2 & 2 & 1 & 8 \\ 3 & 3 & 2 & 13 \end{pmatrix}$ .

## Linear Algebra

2. In the space  $C[0, 1]$  of continuous functions on the interval  $[0, 1]$ , the functions  $e^x$  and  $e^{-x}$  span a two-dimensional subspace. One basis for this subspace, call it the  $E$  basis, is  $[e^x, e^{-x}]$ . Another basis, call it the  $H$  basis, is  $[\cosh(x), \sinh(x)]$ , where the so-called hyperbolic functions are defined as follows:

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad \text{and} \quad \sinh(x) = \frac{e^x - e^{-x}}{2}.$$

Find the transition matrix  $A$  from the  $H$  basis to the  $E$  basis. In other words, find the  $2 \times 2$  matrix  $A$  with the property that if

$$\begin{aligned} f(x) &= a \cosh(x) + b \sinh(x) \\ &= ce^x + de^{-x}, \end{aligned}$$

$$\text{then } A \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix}.$$