Linear Algebra

Write your **name**: ______________________________ (2 points).

In **problems 1–5**, circle the correct answer. (5 points per problem)

1. When A and B are matrices of size 4×6, the sum matrix $A + B$ is always equal to $B + A$. `True` `False`

2. If A is an invertible $n \times n$ matrix, then $\det(A^{-1}) = \dfrac{1}{\det(A)}$.
   `True` `False`

3. For every $n \times n$ matrix A, the product matrix $A^T A$ is a symmetric matrix. `True` `False`

4. If A is a 4×3 matrix, and one of the columns of A has all its entries equal to 0, then the homogeneous linear system $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions. `True` `False`

5. The polynomials $1 + x$, $1 - x$, and $x + x^2$ form a spanning set for the vector space P_3 (which consists of polynomials of degree less than or equal to 2). `True` `False`

In **problems 6–9**, fill in the blanks. (7 points per problem)

6. If $A = \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 0 \\ \boxed{} & 0 & 1 \end{pmatrix}$, then $A^{-1} = \begin{pmatrix} 1 & 0 & \boxed{} \\ 0 & 1 & 0 \\ \boxed{} & 0 & 1 \end{pmatrix}$.

7. If $A = \begin{pmatrix} \boxed{} & 5 \\ \boxed{} & \boxed{} \end{pmatrix}$, then $A + A^T = \begin{pmatrix} 0 & 7 \\ \boxed{} & 0 \end{pmatrix}$.

8. A linearly independent spanning set for a vector space is called a

   ______________________________ .

9. The linear system $\begin{cases} x_1 - 2x_2 + 3x_3 - 4x_4 = 5 \\ 2x_1 - 4x_2 + 6x_3 - 8x_4 = 10 \end{cases}$ has how many

   solutions? ______________________________
   (none, one, or infinitely many?)

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In **problems 10–12**, show your work and explain your method.
(15 points per problem)

10. Determine the null space of the matrix $\begin{pmatrix} 0 & 2 & 1 & 8 \\ 2 & 0 & 1 & 0 \end{pmatrix}$.

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11. Determine a value of x for which

$$\det \begin{pmatrix} 0 & 0 & 0 & x \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -x & 0 & 0 & 8 \end{pmatrix} = 18.$$

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12. The equation below shows an LU factorization. Fill in the missing entries, and explain your strategy for finding them.

$$\begin{pmatrix} \boxed{} & 5 & 0 \\ 3 & \boxed{} & 2 \\ 0 & 4 & \boxed{} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & \boxed{} & \boxed{} \\ 0 & 1 & \boxed{} \\ 0 & 0 & 1 \end{pmatrix}$$