Linear Algebra

Write your **name**: ________________________ (2 points).

In **problems 1–5**, circle the correct answer. (5 points per problem)

1. The equation $L(A) = A - A^T$ defines a linear operator L on the vector space of $n \times n$ matrices. `True` `False`

2. A 3×5 matrix B always has the same rank as the 5×3 matrix B^T. `True` `False`

3. If the linear system $A\mathbf{x} = \mathbf{b}$ is consistent, then the vector $\mathbf{b}$ must belong to the null space of A^T. `True` `False`

4. The transition matrix corresponding to a change of basis in R^n must be an invertible matrix. `True` `False`

5. If the matrix representing a linear transformation $L\colon R^n \to R^n$ with respect to the standard basis has a row of zeroes, then one of the standard basis vectors belongs to the kernel of L. `True` `False`

In **problems 6–9**, fill in the blanks. (7 points per problem)

6. If A is a 30×4 matrix of rank 4, then $\dim N(A)$, the dimension of the null space of A, equals ____________ .

7. If L is the linear operator on R^2 that first reflects in the x-axis and then rotates by 45° counterclockwise, then the matrix representation of L (with respect to the standard basis) is

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \square \\ \square & \square \end{pmatrix}$$

8. If A and B are $n \times n$ matrices, and there exists a nonsingular matrix S such that $B = S^{-1}AS$, then the matrices A and B are called

    ________________________ .

9. In R^2, the vector projection of $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ onto $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ is the vector $\begin{pmatrix} \square \\ \square \end{pmatrix}$.

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In **problems 10–12**, show your work and explain your method.
(15 points per problem)

10. Suppose $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$. Determine a basis for $R(A^T)$, the range of the transpose of A.

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11. Find the distance in the x–y plane from the point $(4, 1)$ to the line with equation $20x + 10y = 0$.

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12. Suppose $\mathbf{u}_1 = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ and $\mathbf{u}_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, and let $\mathbf{e}_1$ and $\mathbf{e}_2$ denote the two standard basis vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Suppose that a linear transformation $L\colon R^2 \to R^2$ is represented with respect to the basis $[\mathbf{u}_1, \mathbf{u}_2]$ by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$. Find the matrix representation of L with respect to the standard basis $[\mathbf{e}_1, \mathbf{e}_2]$.