Write your **name**: ______ (2 points). In **problems 1–5**, circle the correct answer. (5 points per problem)

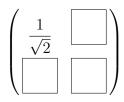
- 1. The equation $L(A) = A A^T$ defines a linear operator L on the vector space of $n \times n$ matrices. True False
- 2. A 3×5 matrix *B* always has the same rank as the 5×3 matrix B^T . True False
- 3. If the linear system $A\mathbf{x} = \mathbf{b}$ is consistent, then the vector \mathbf{b} must belong to the null space of A^T . True False
- 4. The transition matrix corresponding to a change of basis in \mathbb{R}^n must be an invertible matrix. True False
- 5. If the matrix representing a linear transformation $L: \mathbb{R}^n \to \mathbb{R}^n$ with respect to the standard basis has a row of zeroes, then one of the standard basis vectors belongs to the kernel of L. True False

In problems 6–9, fill in the blanks. (7 points per problem)

6. If A is a 30×4 matrix of rank 4, then dim N(A), the dimension of the

null space of A, equals _____ .

7. If L is the linear operator on R^2 that first reflects in the x-axis and then rotates by 45° counterclockwise, then the matrix representation of L (with respect to the standard basis) is



8. If A and B are $n \times n$ matrices, and there exists a nonsingular matrix S such that $B = S^{-1}AS$, then the matrices A and B are called

In **problems 10–12**, show your work and explain your method. (15 points per problem)

10. Suppose $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$. Determine a basis for $R(A^T)$, the range of the transpose of A.

11. Find the distance in the x-y plane from the point (4, 1) to the line with equation 20x + 10y = 0.

12. Suppose $\mathbf{u}_1 = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ and $\mathbf{u}_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, and let \mathbf{e}_1 and \mathbf{e}_2 denote the two standard basis vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Suppose that a linear transformation $L: \mathbb{R}^2 \to \mathbb{R}^2$ is represented with respect to the basis $[\mathbf{u}_1, \mathbf{u}_2]$ by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$. Find the matrix representation of L with respect to the standard basis $[\mathbf{e}_1, \mathbf{e}_2]$.