- There are several cases in which an explicit solution or an implicit solution of a differential equation $\dfrac{dy}{dx} = f(x,y)$ can be computed easily.
  - If the function $f(x,y)$ is a product of a function of x times a function of y, then the differential equation is called *separable*. The equation can be rewritten in the form $g(x)\,dx = h(y)\,dy$, and then integrating both sides gives an implicit solution to the differential equation.
  - If the function $f(x,y)$ depends linearly on y, then the differential equation is called *linear*. The equation can be rewritten in the form $\dfrac{dy}{dx} + P(x)y = Q(x)$. Multiplying both sides of the equation by the *integrating factor* $\mu(x) = \exp(\int P(x)\,dx)$ converts it to the form $\dfrac{d}{dx}\left(\mu(x)y\right) = \mu(x)Q(x)$. Integrating both sides leads to an explicit solution of the differential equation.
    (There is no magic about the form of the integrating factor μ. If you forget the formula, just multiply both sides of the equation by an unknown μ and figure out what μ has to be to make the new left-hand side have the form $(\mu y)'$.)
  - Written in the form $M(x,y)\,dx + N(x,y)\,dy = 0$, a differential equation is called *exact* if the left-hand side is an exact differential, which means that it is of the form dF for some function F. Since $dF = \dfrac{\partial F}{dx}\,dx + \dfrac{\partial F}{dy}\,dy$, the unknown function F must satisfy the two equations $\dfrac{\partial F}{dx} = M(x,y)$ and $\dfrac{\partial F}{dy} = N(x,y)$, so you can find F by two successive integrations. An implicit solution to the differential equation is then $F(x,y) = \text{constant}$.
    Not every differential is an exact differential. A necessary condition for exactness is that $\dfrac{\partial M}{dy} = \dfrac{\partial N}{dx}$, which expresses that the mixed second partial derivatives of F are the same in either order.
- Sometimes a first order differential equation that is not linear, not separable, and not exact can nonetheless be transformed into one of these types by a change of variables.