Math 311-102

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Reminder

The comprehensive final exam is 1:00–3:00PM, Tuesday, July 5, in this room.

Please bring paper (or a bluebook) to the exam.

The exam covers everything on the syllabus.

There are 10 questions on the exam.

Review problems: vector calculus

- Evaluate the line integral $\int_C \vec{F} \cdot d\vec{x}$ when $\vec{F}(x, y, z) = (yz, xz, xy)$ and C is the curve described parametrically by $g(t) = (\cos(t), 1 + \sin(t), t)$, $0 \le t \le 2\pi$.
- Evaluate the surface integral $\iint_S z^2 \, d\sigma$ when S is the surface described by $z = \sqrt{1 - x^2 - y^2}$, $z \ge 0$.
- Evaluate the volume integral $\iiint_B x^2 \, dx \, dy \, dz$ when B is the ball described by $x^2 + y^2 + z^2 \le 1$.
- Evaluate the flux integral $\iint_S \vec{F} \cdot \vec{n} \, d\sigma$ when $\vec{F}(x, y, z) = (y^2, z^2, x^2)$ and S is the open surface described parametrically by $g(u, v) = (u, v, 1 - u^2 - v^2)$, $u^2 + v^2 < 1$, with normal vector oriented upward.

Review problems: vector calculus

- Consider the unit cube in $\mathbb{R}^3$ determined by the three vectors $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$. Orient the surface with the outward-pointing normal vector. Let S be the union of the five faces of the cube on which $z > 0$. Evaluate the integral $\iint_S \operatorname{curl} \vec{F} \cdot \vec{n} \, d\sigma$ when $\vec{F}(x, y, z) = (y^2, z^2, x^2)$.
- Find a point (a, b, c) on the curve $f(t) = (e^t, e^{2t}, e^{3t})$ and a point (d, e, f) on the surface $g(u, v) = (u, v, u^2 + v^2)$ such that the tangent line to the curve at (a, b, c) does not intersect the tangent plane to the surface at (d, e, f).
- Construct an invertible coordinate transformation $\begin{pmatrix} x \\ y \end{pmatrix} = g \begin{pmatrix} u \\ v \end{pmatrix}$ in some region of $\mathbb{R}^2$ such that the area element transforms via $dx \, dy = e^{u+v} \, du \, dv$.

Review problems: linear algebra

- If $A = \begin{pmatrix} -34 & -16 & 36 \\ -18 & -6 & 18 \\ -47 & -21 & 49 \end{pmatrix}$, find a 3×2 matrix B such that $AB = B\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$.
- Suppose $\vec{u}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\vec{u}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\vec{v}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, and $\vec{v}_2 = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$. Find a matrix B such that if $a_1\vec{u}_1 + a_2\vec{u}_2 = b_1\vec{v}_1 + b_2\vec{v}_2$, then $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = B\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$.
- True or false? If V_1 and V_2 are two subspaces of $\mathbb{R}^3$, both of dimension 2, then there exists a linear transformation $f\colon \mathbb{R}^3 \to \mathbb{R}^3$ such that $f(V_1) = V_2$.

Review problems: linear algebra

- Construct a (non-standard) inner product on $\mathbb{R}^2$ for which the vectors $(1, 0)$ and $(1, 1)$ become orthogonal.
- Give an example of a linear transformation $f\colon \mathbb{R}^3 \to \mathbb{R}^2$ such that the vectors $(1, 0)$ and $(1, 1)$ form a basis for the image and the vector $(2, 4, 2)$ is a basis for the null space.
- In the vector space $\mathcal{P}$ of polynomials, express x as a linear combination of $p_1(x) = 1 + x + x^2$, $p_2(x) = 1 + 2x + 3x^2$, and $p_3(x) = 2 - x + 4x^2$.
- The *trace* of a square matrix is the sum of the elements on the main diagonal: for example, $\operatorname{trace}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = 1 + 5 + 9 = 15$. On the vector space of 3×3 matrices, is the trace a *linear* function?