Topics in Applied Mathematics I

We did these exercises in groups.

1. (Exercise 30 on page 45)

   (a) The speed is the length of the velocity vector $10\vec{\imath} + 20\vec{\jmath}$: namely, $10\sqrt{5}$.

   (b) Two sides of the triangle are $\vec{u} = (-2, 2, 0) - (0, 0, 5) = (-2, 2, -5)$ and $\vec{v} = (3, -4, 0) - (0, 0, 5) = (3, -4, -5)$. Then $\frac{1}{2}\vec{v} \times \vec{u}$, which equals $(15, 25/2, -1)$, is a vector perpendicular to the triangle with length equal to the area of the triangle. The flux of air through the triangle equals the dot product of the velocity vector $(10, 20, 0)$ with the vector $(15, 25/2, -1)$: namely, 400.

2. (Exercises 18 and 20 on page 100)

   18. If A^2 equals the zero matrix, it need not be the case that A equals the zero matrix. For example, $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

   20. If $A^2 = O$, then $(I - A)(I + A) = I^2 - A^2 = I$. Hence $(I + A)^{-1} = (I - A)$.

3. (Exercise 38 on page 100)

   If $p(x) = a + bx + cx^2$, then the equations $p(-1) = r$, $p(0) = s$, and $p(1) = t$ translate to the matrix equation $\begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} r \\ s \\ t \end{pmatrix}$. The matrix $\begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ is invertible with inverse $\begin{pmatrix} 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & -1 & \frac{1}{2} \end{pmatrix}$, so there is a unique vector (a, b, c) for every prescribed vector (r, s, t): $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & -1 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} r \\ s \\ t \end{pmatrix}$. In particular, when $(r, s, t) = (1, 0, 1)$, we get $(a, b, c) = (0, 0, 1)$, corresponding to the polynomial x^2.

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4. (Exercises 38 and 40 on page 119)

   38. A subspace always contains the vector $\vec{0}$. If a subspace contains more than one vector, then the subspace contains some non-zero vector $\vec{v}$, hence all scalar multiples of $\vec{v}$. Those scalar multiples describe a line through the origin.

   40. Every subspace containing a non-zero vector contains a line (by the preceding problem), hence contains some vectors of lengths exceeding 1.

       There is, however, one subspace that does satisfy the property that $|\vec{x}| \leq 1$ for all $\vec{x}$ in the subspace: namely, the subspace consisting of just the vector $\vec{0}$.