Math 312 Dr. Boas Spring 1997

Exam 1, February 11

This is a personalized examination. Write down the last four digits of your student identification number and assign the corresponding numbers to the Greek letters α, β, γ, and δ.

α	β	γ	δ

1. Solve the heat equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \qquad 0 < x < \pi, \quad 0 < t,$$

with the boundary conditions $u(0,t) = \alpha$ and $u(\pi, t) = \beta$ and the initial condition $u(x,0) = \gamma x + \delta$, where α, β, γ, and δ are the constants you identified in the table above.

2. A (reasonable) function f that has period 2π has an associated Fourier series

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx).$$

You are used to finding the Fourier coefficients if you know the function. This problem addresses the *inverse problem* of finding the function given the Fourier coefficients.

Construct a function f (with period 2π) that has all of the following properties:

(a) $f(x) = \alpha$ when $-\pi < x < 0$;

(b) the Fourier sine coefficient b_1 is equal to β;

(c) the Fourier cosine coefficient a_γ is equal to δ;

where α, β, γ, and δ are the constants you identified in the table above.