

Linear Algebra

Instructions Please answer the five problems on your own paper. These are essay questions: you should write in complete sentences.

1. Jordan is using a TI-89 calculator to help analyze the linear system $A\mathbf{x} = \mathbf{b}$, where A is a certain 3×4 matrix and \mathbf{b} is a certain 3×1 matrix (a column vector). Jordan applies the `rref` command to the augmented coefficient matrix and obtains the result

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 4 \\ 0 & 0 & 1 & 3 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(Jordan's calculator does not show a vertical bar to separate the last column from the coefficient matrix.) Discuss what information Jordan can deduce about the original linear system.

[For instance, is the system underdetermined or overdetermined? consistent or inconsistent? Is there a unique solution? Does Jordan have enough information to write down the solution(s)?]

2. Consider the system of three simultaneous equations

$$\begin{cases} x_1 + x_2 & = 2 \\ ax_1 + ax_2 & = 3a \\ bx_1 + bx_2 + ax_3 & = 4 + b \end{cases}$$

for the unknowns x_1 , x_2 , and x_3 . Find all values of a and b for which this system of equations is consistent.

Explain your reasoning in complete sentences.

3. Suppose

$$A = \begin{bmatrix} 0 & a & 1 \\ 1 & 0 & 1 \\ 0 & 0 & a \end{bmatrix}.$$

Determine the value(s) of a for which the matrix A is invertible.

[If you do a computation to solve this problem, say what computation you are doing and why.]

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4. Suppose that A is an $n \times n$ matrix, and S is an invertible $n \times n$ matrix. Show that $\det(S^{-1}AS) = \det(A)$.

5. Maude is studying the set of all polynomials in x of odd degree. Help Maude decide if this *set* forms a *vector space* (under the usual operations of addition and scalar multiplication).