Complex Variables

Instructions Please write your name in the upper right-hand corner of the page. Write complete sentences to explain your solutions.

1. Use Cauchy's integral formula to evaluate $\displaystyle\int_{|z-1|=1} \frac{\cos(2\pi z)}{z^2-1}\,dz$, where the integration path is oriented in the standard counterclockwise direction.

2. Student Kan Toor says that if γ is any smooth curve starting at the point 1 and ending at the point i (and not passing through 0), then

$$\int_\gamma \frac{1}{z}\,dz = \log(i) - \log(1) = \frac{i\pi}{2}.$$

"I know there is an ambiguity in the value of the logarithm, but since I am taking a difference of two values, the ambiguity cancels out," says Kan Toor. Is Kan Toor's analysis correct? Explain why or why not.

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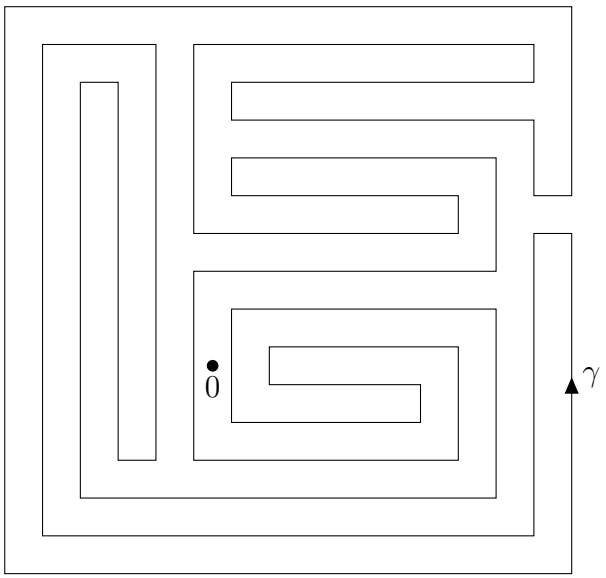
3. Student Ko Shi says that $\displaystyle\int_\gamma \frac{1}{z}\,dz = 2\pi i$, where γ is the simple closed curve indicated below.



Is Ko Shi right or wrong? Explain why.