1. Evaluate $\int_{C_{R}} \frac{e^{i z}}{\left(z^{2}+1\right)^{2}} d z$ by using the residue theorem (notice the double pole at $i$ ). Deduce that

$$
\int_{0}^{\infty} \frac{\cos (x)}{\left(x^{2}+1\right)^{2}} d x=\frac{\pi}{2 e}
$$

2. Show that $\lim _{N \rightarrow \infty} \int_{C_{N}} \frac{\pi}{z^{2} \sin (\pi z)} d z=0$ (where $N$ runs through the natural numbers). Then evaluate the integral by using the residue theorem. Deduce that



$$
\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{2}}=\frac{\pi^{2}}{12}
$$

