Complex Variables Quiz

1. Evaluate $\int_{C_R} \frac{e^{iz}}{(z^2+1)^2} dz$ by using the residue theorem (notice the double pole at *i*). Deduce that

$$\int_0^\infty \frac{\cos(x)}{(x^2+1)^2} \, dx = \frac{\pi}{2e}.$$

2. Show that $\lim_{N \to \infty} \int_{C_N} \frac{\pi}{z^2 \sin(\pi z)} dz = 0$ (where *N* runs through the natural numbers). Then evaluate the integral by using the residue theorem. Deduce that

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}.$$

