1. Define each of the following terms:
   (a) continuous function;
   (b) Lipschitz continuous function;
   (c) Riemann integrable function.

2. State and prove *either* the mean value theorem *or* Taylor's theorem.

3. (a) State the definition of the derivative $f'(x)$ in terms of a limit.
   (b) Use this definition to derive the product rule: namely, if f and g are differentiable functions, then $(fg)' = f'g + g'f$.

4. The sum $\displaystyle\sum_{j=0}^{n-1} \frac{2n}{n^2 + j^2}$ can be interpreted as an upper sum for a certain continuous function on a certain interval. Specify the function f, the interval $[a, b]$, and the partition of the interval.

5. Give a concrete example of each of the following:
   (a) a function f that *is* Riemann integrable on the interval $[0, 1]$ even though f is *not* continuous on $[0, 1]$;
   (b) a function g that is *not* Riemann integrable on the interval $[0, 1]$ even though g *is* bounded on $[0, 1]$;
   (c) a function h such that $\displaystyle\int_0^1 h(x)\,dx$ *does* exist as an *improper* Riemann integral even though it does *not* exist as an ordinary Riemann integral.

6. (a) State the fundamental theorem of calculus. (It connects the theory of differentiation to the theory of integration.)
   (b) State the chain rule. (It tells how to differentiate a composite function.)
   (c) Apply these two theorems to compute the derivative $F'(x)$ of the function F that is defined by the formula $F(x) := \displaystyle\int_0^{1/x} e^{t^2}\,dt$.