

1. For the three definitions, see pages 73, 85, and 89.
2. See Theorem 4.2.3 on page 130 and Theorem 4.3.1 on page 136.
3. In view of the definition on page 121, we may write

$$\begin{aligned}
 (fg)'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \\
 &= \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \cdot g(x+h) + \frac{g(x+h) - g(x)}{h} \cdot f(x) \right) \\
 &= f'(x)g(x) + g'(x)f(x).
 \end{aligned}$$

We did this calculation in class. The last step uses that the limit of a product equals the product of the limits.

4. Rewriting $\sum_{j=0}^{n-1} \frac{2n}{n^2 + j^2}$ as $\sum_{j=0}^{n-1} \frac{1}{n} \cdot \frac{2}{1 + \left(\frac{j}{n}\right)^2}$ makes it possible to recognize this sum as an upper sum for the function $\frac{2}{1+x^2}$ on the interval $[0, 1]$ with the partition $0, 1/n, 2/n, \dots, n/n$. Since the function is decreasing, the upper sum is identical to the left-hand-endpoint sum.
5. There are many possible examples. Example 1 on page 103 solves part (b), Example 1 on page 113 solves part (c), and one solution for part (a) is a step function like $f(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ 1, & \frac{1}{2} < x \leq 1. \end{cases}$
6. Parts (a) and (b) are Theorems 4.2.4/4.2.5 on pages 131–132 and Theorem 4.1.3 on page 125. For part (c), view F as $G \circ g$, where $G(u) = \int_0^u e^{t^2} dt$ and $g(x) = 1/x$. Then $F'(x) = G'(g(x))g'(x)$ by the chain rule, while $G'(u) = e^{u^2}$ by the fundamental theorem of calculus, and $g'(x) = -1/x^2$. Hence

$$F'(x) = -\frac{1}{x^2} \exp\left(\frac{1}{x^2}\right).$$