Math 409 **Advanced Calculus I** Fall 2004

First Examination

1. (a) i. Define what it means for a sequence of real numbers to be monotone.
   ii. Give an example of a monotone sequence.
   iii. Give an example of a sequence that is not monotone.

   (b) i. Define what it means for a sequence of real numbers to be a Cauchy sequence.
   ii. Give an example of a Cauchy sequence.
   iii. Give an example of a sequence that is not a Cauchy sequence.

2. (a) State the Bolzano-Weierstrass theorem.

   (b) State the squeeze theorem about limits of sequences.

3. (a) State the definition of "the limit of the sequence $\{a_n\}$ is L".

   (b) Use the definition in part (a) to prove that 0 is the limit of the sequence $\{1/\sqrt{n^2+1}\,\}$.

4. For each of the following statements, say whether the statement is true or false. To support an answer of "true", you must give an explanation or cite a theorem or supply a proof; to support an answer of "false", you must exhibit a counterexample.

   (a) If a sequence $\{a_n\}$ is unbounded, then the sequence $\{a_n\}$ has no cluster point.

   (b) If a sequence $\{a_n\}$ converges, then the related sequence $\{(-1)^n a_n\}$ diverges.

5. Prove that the infinite series $\sum_{n=1}^{\infty} 1/\sqrt{n^2+1}$ diverges.
   (Notice that this problem differs from problem 3(b): that problem concerns a *sequence*, but this problem concerns a *series*.)