Math 409-502

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Announcement

TAMU Math Club Meeting

Monday, September 20

7:00pm in Blocker 627

Speakers:
Dr. Philip Yasskin, “Rascal’s Triangle”
Ms. Edith Andrews, Jane Long Middle School HOSTS program

FREE FOOD

More about subsequences and convergence

Main theorems in Chapter 6

- Nested interval theorem
- Bolzano-Weierstrass theorem

Main concepts in Chapter 6

- cluster point
- Cauchy sequence
- supremum
- $\limsup$

Nested intervals

Theorem. *If the* closed *intervals*

$$[a_1, b_1] \supseteq [a_2, b_2] \supseteq \cdots \supseteq [a_n, b_n] \supseteq \ldots$$

are nested, then the intersection $\bigcap_{n=1}^{\infty} [a_n, b_n]$ *is not empty.*
Moreover, if length$[a_n, b_n] \to 0$, *then there is* exactly one point *common to all the intervals.*

Examples

- The nested intervals $[-1 - 1/n, 1 + 1/n]$ have intersection $[-1, 1]$.
- The nested intervals $[1 - 1/n, 1]$ have intersection $\{1\}$.
- The nested *open* intervals $(0, 1/n)$ have *empty* intersection.

Bolzano-Weierstrass theorem

Theorem. *A bounded sequence of real numbers has convergent subsequences.*

Proof: repeated bisection and the nested interval theorem.

Examples

- The sequence $\{\sin n\}_{n=1}^{\infty}$ has convergent subsequences.
- Let x_n be the right-most digit of the nth prime number. Then the sequence $\{x_n\}_{n=1}^{\infty}$ has convergent subsequences.

Cluster points

Definition

A *cluster point* of a sequence is the limit of a convergent subsequence. (Another name for the same concept is *accumulation point*.)

Examples

- The sequence $\{(-1)^n\}_{n=1}^{\infty}$ has two cluster points: namely 1 and -1.
- The sequence $\{n\sin(n\pi/2)\}_{n=1}^{\infty}$ has one cluster point: namely 0.

Homework

- Read sections 6.1–6.3, pages 78–83.
- Do Exercises 6.2/1 and 6.3/1 on pages 89–90.