

Please write your solutions on your own paper. These problems should be treated as essay questions to answer in complete sentences.

1. Show that the positive values of  $x$  for which  $x \log(5) = 5 \log(x)$  are like the Sith Lords in *Star Wars*: “two there are—no more, no less.”
2. Consider the sequence  $\{x_n\}$  defined recursively as follows:

$$x_1 = 5, \quad \text{and} \quad x_{n+1} = 5 - \frac{1}{x_n} \quad \text{when } n \geq 1.$$

Does this sequence converge? Why or why not?

3. Let  $E$  denote the subset of the interval  $[0.05, 0.55]$  consisting of real numbers that can be written as decimal expansions using only the digits 0 and 5. Explain why  $E$  is a compact set with empty interior.

4. Show that  $\lim_{n \rightarrow \infty} \int_0^5 x^{1/2} \cos(nx) dx = 0$ .

5. a) State the completeness axiom for the real numbers.  
b) Define the concept of continuity of a function at a point.

6. Show that  $\lim_{x \rightarrow 0} \frac{(\sin x)^5 - x^5}{(1 - \cos x) \sin(x^5)} = -\frac{5}{3}$ .

**Extra Credit.** The number  $\Gamma(1/5)$ , also known as  $5 \cdot (\frac{1}{5}!)$ , can be expressed as the convergent improper integral

$$\int_0^{\infty} x^{-4/5} e^{-x} dx.$$

Show that the numerical value of  $\Gamma(1/5)$  lies between 4 and 5.

Hint: Consult the captured secret plans below.

