Reminder

Exam 1 on Chapters 1–3 takes place in class on Thursday, February 23.

Please bring your own paper to work on.

Properties of the real numbers

- $\mathbb{R}$ is a complete ordered field.
- Archimedean property: $\mathbb{N}$ is not bounded above in $\mathbb{R}$.
- $\mathbb{Q}$ is dense in $\mathbb{R}$.

Sequences

- Null sequence
- Convergent sequence
- Cauchy sequence

In $\mathbb{R}$, convergent sequences and Cauchy sequences are the same. In $\mathbb{Q}$, which lacks completeness, convergent sequences are Cauchy sequences, but Cauchy sequences need not converge to an element of $\mathbb{Q}$.

Convergence theorems

- Monotone convergence
- Squeeze theorem
- Cantor's nested interval theorem

Subsequences

- Bolzano–Weierstrass theorem
- $\limsup$ and $\liminf$.
  $\lim x_n$ exists precisely when $\limsup x_n = \liminf x_n$.
- A property holds *frequently* precisely when the property holds for some subsequence.
- A property holds *ultimately* precisely when the negation does not hold frequently.

What should $\lim_{n \to \infty} x_n = \infty$ mean?
For every N, the sequence is ultimately greater than N.
$\forall N \ \exists C$ such that $x_n > N$ when $n \geq C$.

For a sequence of sets (A_n), define the $\limsup$ to be

$$\bigcap_{n=1}^{\infty}\left(\bigcup_{k\geq n} A_k\right)$$

x is this intersection if for every n,

$$x \in \bigcup_{k\geq n} A_k$$

which happens if there exists some k greater than or equal to n for which $x \in A_k$.
That is, x is frequently in A_n.