Questions

Induction: Want to prove some statement $P(n)$ for all natural numbers n.

Strategy:

1. basis step: prove $P(1)$
2. induction step: prove the implication $\forall n$ $(P(n) \implies P(n+1))$

Equivalent statement is the well-ordering property of $\mathbb{N}$: namely, every nonempty set of natural numbers has a least element.

A proof that $\sqrt{2}$ is irrational via the well-ordering property, after Richard Dedekind (1831–1916)

Proof by contradiction.

If $\sqrt{2}$ is a rational number, then there is a natural number n such that $n\sqrt{2}$ is an integer. By the well-ordering property of $\mathbb{N}$, there is a least such n, say k. Let j denote the difference $k\sqrt{2} - k$. Then

(a) j is an integer (the difference of two integers)

(b) $0 < j < k$ (by a calculation)

(c) $j\sqrt{2}$ is an integer (namely, $2k - k\sqrt{2}$)

Therefore k is not least after all. Contradiction. The contradiction means that $\sqrt{2}$ cannot be a rational number. $\square$

Assignment due next class

- Write solutions to Exercises 0.3.19 and 0.3.20.
- Read section 1.1 in the textbook.