Warm-up exercise

Suppose A and B are bounded sets of real numbers.

1. If $C = A \cup B$ (the union), then what can you say about $\sup(C)$ in terms of $\sup(A)$ and $\sup(B)$?
   Answer: $\sup(C) = \max\{\sup(A), \sup(B)\}$

2. If $D = A \cap B$ (the intersection), then what can you say about $\sup(D)$ in terms of $\sup(A)$ and $\sup(B)$?
   Answer: $\sup(A \cap B) \leq \min\{\sup(A), \sup(B)\}$, but equality does not necessarily hold.

3. If $E = A \setminus B$ (the complement), then what can you say about $\sup(E)$ in terms of $\sup(A)$ and $\sup(B)$?
   Answer: $\sup(A \setminus B) \leq \sup(A)$, but equality does not necessarily hold.

Example of a non-Archimedean ordered field

The rational functions (ratios of polynomials),
objects like $\frac{5x^3 + \pi x^2 - \frac{22}{7}}{2x^2 - \sqrt{8}}$,
form a field.

This field can be *ordered* by declaring a rational function to be "positive" when the ratio of leading coefficients is positive.

In this field, the element x (the rational function $x/1$) is not only positive but "infinitely large" (greater than every natural number).

In this field, the subset $\mathbb{N}$ is bounded above but has no least upper bound.

Assignment due next class

- Write a solution to Exercise 1.2.1.
- Read section 1.3 in the textbook.