Reminder

- The second exam takes place on Wednesday, March 28.
- Material covered: sections 2.3, 2.4, 2.5, 2.6.1, 2.6.2, 3.1.

Yet another characterization of cluster points

A point p is a cluster point of a set S when every neighborhood of p contains infinitely many points of S.

Examples

Determine the cluster points of the following sets.

1. $B = \left\{ \dfrac{1}{p^2} : p \text{ is a prime number} \right\}$.
   The only cluster point is 0.

2. $A = \mathbb{Q} \cap (0, 1)$.
   The set of cluster points is the closed interval $[0, 1]$.

3. $C = \{ n(-1)^n : n \in \mathbb{N} \}$.
   No cluster points: the set C is equal to its closure.

Definition: The *closure* of a set is the union of the set with all of its cluster points.

Definition of limit of a function

If $f \colon S \to \mathbb{R}$, and c is a cluster point of S, then

$$\lim_{x \to c} f(x) = L$$

means that $\lim\limits_{n\to\infty} f(x_n) = L$ for every sequence $\{x_n\}_{n=1}^{\infty}$ such that $\lim\limits_{n\to\infty} x_n = c$, and $x_n \in S$ for every n, and $x_n \neq c$ for every n.

Equivalently: for every positive ε, there exists a positive δ such that $|f(x) - L| < \varepsilon$ when $|x - c| < \delta$ and $x \neq c$.

Assignment due next class

- Read the rest of section 3.1.
- Solve your group's part of Exercise 3.1.1 and be prepared to present the solution to the rest of the class.