Exam results

Average 86: great job!

Continuous functions

A function $f\colon S \to \mathbb{R}$ is *continuous at a point c in S* when

- either c is an isolated point of S,
- or c is a cluster point of S, and
  - $\lim\limits_{x \to c} f(x)$ exists, and
  - $\lim\limits_{x \to c} f(x) = f(c)$.

Equivalent statement in terms of sequences:
For every sequence $\{x_n\}_{n=1}^{\infty}$ of points of S such that $\lim\limits_{n \to \infty} x_n = c$, the image sequence $\{f(x_n)\}_{n=1}^{\infty}$ converges to $f(c)$.

Some examples

- Since limits of sequences preserve sums and products, every polynomial function is continuous at every real number.
- Similarly, rational functions (ratios of polynomials) are continuous at points where the denominator is nonzero.
- The sine function and the cosine function are continuous at every real number.
  (The book sketches a proof by using trigonometric identities.)
- The exponential function e^x.

A function operation that has no sequence analogue

The composite function $f \circ g$ sends the input x to the output $f(g(x))$.

Example: $\cos(x^2)$.

Composing two continuous functions produces another continuous function.

Assignment due next class

- Read section 3.2 in the textbook.
- Find two functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$, each of which has a point of discontinuity, yet the composite functions $f \circ g$ and $g \circ f$ are everywhere continuous.