1. Let $R$ denote the region in the first quadrant of the $x-y$ plane bounded by the line $y=x$ and the parabola $y=x^{2}$. Compute
(a) the area of $R$,
(b) the center of gravity of $R$.
2. Rewrite $\int_{-1}^{2} d x \int_{x}^{x^{3}} f(x, y) d y$ as an iterated integral in the other order.
3. Let $R$ denote the closed unit square: the region in the first quadrant of the $x-y$ plane defined by the inequalities $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Give a concrete example
(a) of a function $f$ that is not uniformly continuous on $R$ but that is Riemann integrable on $R$,
(b) of a function $f$ that is defined everywhere on $R$ but that is not Riemann integrable on $R$.
4. Compute the surface area of the piece of the unit sphere $x^{2}+y^{2}+z^{2}=1$ on which all three coordinates $x, y$, and $z$ are positive and in addition $x<1 / 2$.
5. Prove from first principles that if $f$ and $g$ are continuous functions on a compact region $R$, and if $f(x, y) \leq g(x, y)$ for all points $(x, y)$ in $R$, then $\iint_{R} f(x, y) d S \leq \iint_{R} g(x, y) d S$.
6. Model the surface of a pumpkin by the equation $r=1+a(\sin \varphi)|\cos 8 \theta|$ in spherical coordinates, where $a$ is a small positive number. As usual, $r$ is the distance from the origin, the angle $\theta$ is the polar angle, and the angle $\varphi$ is the co-latitude (the angle measured down from the positive $z$-axis). The figure illustrates the case
 $a=1 / 10$.
Set up an integral that expresses the volume of the pumpkin. Do not evaluate the integral.
