- 1. Let R denote the region in the first quadrant of the x-y plane bounded by the line y = x and the parabola  $y = x^2$ . Compute
  - (a) the area of R,
  - (b) the center of gravity of R.
- 2. Rewrite  $\int_{-1}^{2} dx \int_{x}^{x^{3}} f(x, y) dy$  as an iterated integral in the other order.
- 3. Let R denote the closed unit square: the region in the first quadrant of the x-y plane defined by the inequalities  $0 \le x \le 1$  and  $0 \le y \le 1$ . Give a concrete example
  - (a) of a function f that is not uniformly continuous on R but that is Riemann integrable on R,
  - (b) of a function f that is defined everywhere on R but that is not Riemann integrable on R.
- 4. Compute the surface area of the piece of the unit sphere  $x^2 + y^2 + z^2 = 1$ on which all three coordinates x, y, and z are positive and in addition x < 1/2.
- 5. Prove from first principles that if f and g are continuous functions on a compact region R, and if  $f(x, y) \leq g(x, y)$  for all points (x, y) in R, then  $\iint_R f(x, y) dS \leq \iint_R g(x, y) dS$ .
- 6. Model the surface of a pumpkin by the equation  $r = 1 + a(\sin \varphi) |\cos 8\theta|$  in spherical coordinates, where a is a small positive number. As usual, r is the distance from the origin, the angle  $\theta$  is the polar angle, and the angle  $\varphi$  is the co-latitude (the angle measured down from the positive z-axis). The figure illustrates the case a = 1/10.



Set up an integral that expresses the volume of the pumpkin. Do not evaluate the integral.