Math 433 Exam 1 Summer 2009

Applied Algebra

Instructions Please answer these questions on your own paper. Explain your work in complete sentences.

1. Determine the smallest positive integer n with the property that there exist integers x and y such that $60x + 42y = n$.

2. Prove by induction that

$$(1! \cdot 1) + (2! \cdot 2) + \cdots + (n! \cdot n) = (n+1)! - 1$$

   for every positive integer n (where, as usual, the factorial $n!$ means the product of all the integers between 1 and n inclusive).

3. When the number $65^{93} \times 56^{39}$ is written out, it has 237 digits. How many zeroes are there at the right-hand end? Explain how you know.

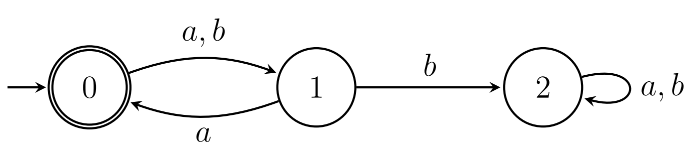
4. Find a multiplicative inverse of 23 modulo 31.

5. Solve the pair of simultaneous linear congruences

$$\begin{cases} x \equiv 6 \pmod{7}, \\ x \equiv 5 \pmod{17}. \end{cases}$$

6. Using the RSA system, I encoded my birthday (month and day) in two blocks as 30 5. The public key is the pair $(33, 7)$, where 33 is the base n and 7 is the exponent a. When is my birthday?

7. Describe the words (sequences of letters a and b) that the following finite-state automaton accepts.



8. Let R be the relation defined on the set of positive integers by xRy if and only if $x^2 \equiv y^3 \pmod 4$. Is this relation R reflexive? symmetric? transitive? Explain how you know.

9. State the Chinese Remainder Theorem.