

Applied Algebra

Instructions Please write your name in the upper right-hand corner of the page. Use complete sentences, along with any necessary supporting calculations, to answer the following questions.

1. You know that \mathbb{Z} (the set of integers) is an abelian group under addition. You know too that \mathbb{Z}_2 (the integers modulo 2) is a field (since the number 2 is prime). Consider the “scalar multiplication” operation of \mathbb{Z}_2 on \mathbb{Z} defined as follows: $[0]_2 \cdot n = 0$ and $[1]_2 \cdot n = n$ for every integer n . Does this operation give \mathbb{Z} the structure of a vector space over the field \mathbb{Z}_2 ? Explain why or why not.

Applied Algebra

2. The two partially completed tables shown are the addition table and the multiplication table for a certain field. The labels 0 and 1 represent the additive identity element and the multiplicative identity element. Fill in the missing entries.

+	0	1	a	b
0	0	1	a	b
1	1	0		
a	a			
b	b			

\cdot	0	1	a	b
0		0		
1	0	1	a	b
a		a		
b		b		