Applied Algebra

Instructions   Please write your name in the upper right-hand corner of the page. Use complete sentences, along with any necessary supporting calculations, to answer the following questions.

1. You know that $\mathbb{Z}$ (the set of integers) is an abelian group under addition. You know too that $\mathbb{Z}_2$ (the integers modulo 2) is a field (since the number 2 is prime). Consider the "scalar multiplication" operation of $\mathbb{Z}_2$ on $\mathbb{Z}$ defined as follows: $[0]_2 \cdot n = 0$ and $[1]_2 \cdot n = n$ for every integer n. Does this operation give $\mathbb{Z}$ the structure of a vector space over the field $\mathbb{Z}_2$? Explain why or why not.

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2. The two partially completed tables shown are the addition table and
   the multiplication table for a certain field. The labels 0 and 1 represent
   the additive identity element and the multiplicative identity element.
   Fill in the missing entries.

+	0	1	a	b
0	0	1	a	b
1	1	0		
a	a			
b	b			

$\cdot$	0	1	a	b
0		0		
1	0	1	a	b
a		a		
b		b		