Applied Algebra

Instructions   Please write your name in the upper right-hand corner of the page. Use complete sentences, along with any necessary supporting calculations, to answer the following questions.

1. Suppose that F is a field in which $(x + y)^2 = x^2 + y^2$ for all elements x and y. Show that F has characteristic 2 (that is, $1 + 1 = 0$ in F).

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2. Consider the set of 2×2 matrices with real entries. Suppose that matrices are added and multiplied in the usual way, but a nonstandard scalar multiplication rule is defined as follows:

$$\lambda \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \lambda a & b \\ \lambda c & d \end{pmatrix} \qquad \text{for every real number } \lambda.$$

Do these operations provide the set of 2×2 matrices with the structure of an algebra (a ring that is simultaneously a vector space)? Explain.