Follow-up on derived sets

Class on February 4 studied how intersection, union, and complement interact with interior, closure, frontier, exterior, and the derived set in a topological space X. Here is a summary.

Interior

Intersection $(A \cap B)^\circ = (A^\circ) \cap (B^\circ)$ for all sets A and B.

Union $(A^\circ) \cup (B^\circ)$ always is a subset of $(A \cup B)^\circ$ and sometimes is a proper subset.

Complement $(X \setminus A)^\circ$ always is a subset of $X \setminus (A^\circ)$ and sometimes is a proper subset.

Closure

Intersection $\mathrm{Cl}(A \cap B)$ always is a subset of $(\mathrm{Cl}\, A) \cap (\mathrm{Cl}\, B)$ and sometimes is a proper subset.

Union $\mathrm{Cl}(A \cup B) = (\mathrm{Cl}\, A) \cup (\mathrm{Cl}\, B)$ for all sets A and B.

Complement $X \setminus (\mathrm{Cl}\, A)$ always is a subset of $\mathrm{Cl}(X \setminus A)$ and sometimes is a proper subset.

Frontier

Intersection $\mathrm{Fr}(A \cap B)$ and $(\mathrm{Fr}\, A) \cap (\mathrm{Fr}\, B)$ have no general relationship to each other.

Union $\mathrm{Fr}(A \cup B)$ always is a subset of $(\mathrm{Fr}\, A) \cup (\mathrm{Fr}\, B)$ and sometimes is a proper subset.

Complement $\mathrm{Fr}(X \setminus A)$ and $X \setminus (\mathrm{Fr}\, A)$ are complements of each other, since $\mathrm{Fr}(X \setminus A) = \mathrm{Fr}\, A$.

Exterior

Intersection $(\mathrm{Ext}\, A) \cap (\mathrm{Ext}\, B)$ always is a subset of $\mathrm{Ext}(A \cap B)$ and sometimes is a proper subset.

Union $\mathrm{Ext}(A \cup B)$ always is a subset of $(\mathrm{Ext}\, A) \cup (\mathrm{Ext}\, B)$ and sometimes is a proper subset.

Complement $\mathrm{Ext}(X \setminus A)$ always is a subset of $X \setminus (\mathrm{Ext}\, A)$ and sometimes is a proper subset.

Derived set

Intersection $(A \cap B)'$ always is a subset of $(A') \cap (B')$ and sometimes is a proper subset.

Union $(A \cup B)' = (A') \cup (B')$ for all sets A and B.

Complement $(X \setminus A)'$ and $X \setminus (A')$ have no general relationship to each other.