

Exercise on topologies

Let X be the set $\{\alpha, \beta, \gamma\}$.

1. Find all possible topologies on X .
2. Find all possible topologies on X for which the only clopen sets are \emptyset and X .
3. Find all possible topologies on X for which some point is open.
4. Find all possible topologies on X for which some point is closed.

In each case, draw a diagram that orders the topologies by coarseness and fineness.