Remark on terminology

The word "separable"
(meaning the existence of a countable dense set)
is unrelated to the separation properties T_0, T_1, T_2, T_3
and also has nothing to do with connectedness.

Maurice Fréchet
(1878–1973)

Fréchet introduced the unfortunate term "separable" in his 1906 dissertation.

Continuity

When (X_1, τ_1) and (X_2, τ_2) are topological spaces, a function $f \colon X_1 \to X_2$ is *continuous* when $f^{-1}(B) \in \tau_1$ whenever $B \in \tau_2$.

"f is continuous when the inverse image of every open set is open."

Example

If $f \colon (\mathbb{R}, \text{Euclidean}) \to (\mathbb{R}, \text{discrete})$ is defined by $f(x) = x^2$, then f is not continuous, because $\{1\}$ is open in the discrete topology, but $f^{-1}(\{1\}) = \{-1, 1\}$, which is not open in the Euclidean topology.
Same f viewed as a function from $(\mathbb{R}, \text{discrete})$ to $(\mathbb{R}, \text{Euclidean})$ is continuous (because every subset of the domain is open).
In fact every function from a discrete space to an arbitrary space is continuous.
Also, every function from a space into an indiscrete space is continuous.

Assignment due next class

- Read section 5.1 in the textbook.
- Write a solution to number 1 in Exercises 5.1.