

Components [Exercise 5.2.9]

A *component* of a topological space is a maximal connected subset.

Example

If $X = \mathbb{R} \setminus \mathbb{Z}$, then the components of X are open intervals of the form $(n, n + 1)$, where n is an integer.

In this example, the components happen to be clopen subsets of X .

Components are *always* closed sets [Exercise 5.2.5].

If there are finitely many components, then each one is open too.

But in general, components need not be open sets.

Example

If $X = \mathbb{Q}$, then the components of X are singletons. In \mathbb{Q} , singletons are closed but not open.

Path components [Exercise 5.2.11]

A *path component* of a topological space is a maximal path-connected subset.

Since path-connected sets are connected, each path component is contained in (but not necessarily equal to) some component.

Example

The path components of the topologist's sine curve are the graph $\{ (x, \sin(\frac{1}{x})) : x > 0 \}$ and the vertical line segment $\{ (0, y) : -1 \leq y \leq 1 \}$.

Notice that one of these path components is open but not closed, and the other component is closed but not open.

Assignment due next class

If X is the natural numbers with the initial-segment topology, then

1. which subsets of X are connected?
2. which subsets of X are path-connected?

Explain why.