

1. Give an example of a family of entire functions that is not a normal family when considered as a family of functions whose domain is the whole plane  $\mathbb{C}$  but that is a normal family when considered as a family of functions whose domain is the unit disc  $\{z \in \mathbb{C} : |z| < 1\}$ .

2. Determine the number of solutions to the equation

$$z^{11} + 25z^7 + 43z = 1$$

in the annulus  $\{z \in \mathbb{C} : 1 < |z| < 2\}$ .

3. In view of the Riemann mapping theorem, there exists an invertible holomorphic function (a conformal mapping) that maps the first quadrant  $\{z \in \mathbb{C} : \operatorname{Re} z > 0 \text{ and } \operatorname{Im} z > 0\}$  onto the unit disc  $\{z \in \mathbb{C} : |z| < 1\}$ . Find an explicit example of such a function.
4. Suppose  $f$  is a holomorphic function that is defined in the unit disc  $\{z \in \mathbb{C} : |z| < 1\}$  and that has image contained in the unit disc. If  $f(0) = 0$  and  $f'(0) = 0$ , how big can  $|f''(0)|$  be?
5. Suppose that  $G$  is a connected open subset of  $\mathbb{C}$ , and  $\{f_n\}_{n=1}^{\infty}$  is a sequence of holomorphic functions converging normally (that is, uniformly on compact subsets of  $G$ ) to a non-constant holomorphic limit function  $f$ . Prove that if every function  $f_n$  in the sequence is a one-to-one (that is, injective) function, then the non-constant limit function  $f$  must be one-to-one.