Math 617

## Final Exam

# Theory of Functions of a Complex Variable I

Here (in alphabetical order) is the list compiled in class of eleven theorems from the semester.

- 1. Cauchy's integral formula
- 2. Cauchy's theorem
- 3. Identity theorem
- 4. Laurent series, existence and uniqueness
- 5. Liouville's theorem
- 6. Maximum principle
- 7. Morera's theorem
- 8. Open mapping theorem
- 9. Residue theorem
- 10. Rouché's theorem
- 11. Schwarz lemma

### Part A

State six of these theorems, including at least three corresponding to prime numbers.

### Part B

Prove four of the theorems, including at least one corresponding to a perfect square.

### Remarks

The theorems that you choose for Part B may be, but are not required to be, a subset of your theorems from Part A.

In the case of a theorem that has more than one version, you may (both in Part A and in Part B) choose whichever version you wish.

### Optional bonus question for extra credit

Two weeks from today (December 29) is the 149th anniversary of the birth of algebraist Kurt Hensel, inventor of the *p*-adic numbers. The great number theorist and analyst Peter Dirichlet was married to Hensel's great-aunt, and her cousin was married to Ernst Kummer, who is responsible for the word "ideal" in algebra. Kummer's son-in-law (who was also Kummer's student) made an appearance in this course. What is his name?