Announcements about next week, November 19–23

Monday	Office hour canceled, but I will be available via email.
Tuesday	Regular class schedule: Math 617 meets as usual.
Wednesday	TAMU classes do not meet ("Reading Day").
Thursday	Thanksgiving Day: TAMU classes do not meet.
Friday	Thanksgiving Holiday continues. TAMU classes do not meet.

Casorati–Weierstrass theorem

The proof of Picard's great theorem is a topic for Math 618, but a weaker theorem is provable now.

Theorem (Casorati–Weierstrass)

In every punctured neighborhood of an essential singularity, the range of the function is dense in $\mathbb{C}$.

Proof.

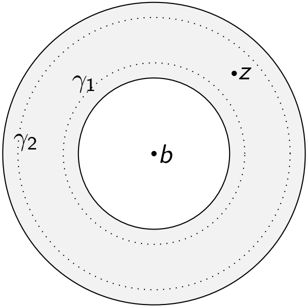
If not, the complement of the range of the function f contains some disk $B(c; r)$. Then $\dfrac{1}{f(z) - c}$ is bounded, hence has a removable singularity at b. So $\displaystyle\lim_{z \to b} \frac{1}{f(z) - c}$ exists.
If the limit is zero, then f has a pole at b.
If the limit is nonzero, then f has a removable singularity at b.
Both cases contradict that f has an essential singularity. □

Analytic functions in annuli

Every function analytic in an annulus is the sum of a function that is analytic inside the outer circle and a function that is analytic outside the inner circle. Why?

The homology form of Cauchy's integral formula implies that

$$f(z) = \frac{1}{2\pi i}\int_{\gamma_2}\frac{f(w)}{w-z}\,dw - \frac{1}{2\pi i}\int_{\gamma_1}\frac{f(w)}{w-z}\,dw.$$



Continuation

The function analytic inside the outer circle admits a series expansion in positive powers of $(z - b)$ (a Taylor series).

The function analytic outside the inner circle admits a series expansion in negative powers of $(z - b)$.

The total series $\sum_{n=-\infty}^{\infty} c_n (z - b)^n$ is a *Laurent series*.

The series converges uniformly on compact subsets of the annulus.

Assignment due next time

- There is no assignment to hand in, but you are welcome to read some of Chapter V of the textbook.