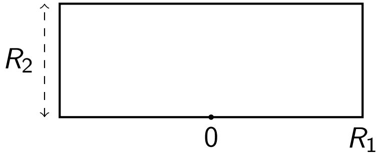
Recap: basic version of the residue theorem

The *residue* of f at an isolated singular point b is the coefficient of $\dfrac{1}{z-b}$ in the Laurent series of f in a punctured disk with center b.

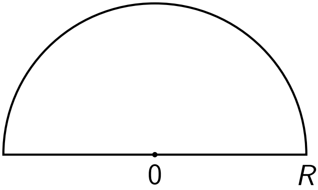
If G is a simply connected open set; and if the singularities of f in G are isolated; and if γ is a simple, closed, counterclockwise, rectifiable curve in G not passing through any singular points; then

$$\frac{1}{2\pi i}\int_\gamma f(z)\,dz = \text{sum of residues of } f \text{ at singularities inside } \gamma.$$

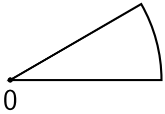
Some standard closed integration contours



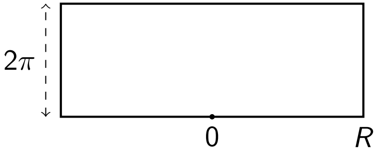
Cauchy's contour



Jordan's contour



piece of pie



Exercise V.2.2(g)

Exercises

(A) $\displaystyle\int_0^{2\pi} \frac{1}{1+(\sin\theta)^2}\,d\theta = \pi\sqrt{2}$. Hint: $z = e^{i\theta}$, $d\theta = (dz)/iz$.

(B) $\displaystyle\int_0^{\infty} \frac{1}{(x^2+1)^4}\,dx = \frac{5\pi}{32}$.

(C) $\displaystyle\int_0^{\infty} \frac{\cos x}{1+x^2}\,dx = \frac{\pi}{2e}$. Hint: $\cos(x) = \mathrm{Re}(e^{ix})$.

(D) $\displaystyle\int_0^{\infty} \frac{\sqrt{x}}{1+x^2}\,dx = \frac{\pi}{\sqrt{2}}$

Assignment

- Find out when the final examination takes place.
- Please complete the online course evaluation.
- Convince yourself that you can compute integrals by applying the residue theorem.