# Exercise on the definition of infinite products

The goal of this exercise is to arrive at a reasonable definition of what it means for an infinite product to converge. I am not telling you a definition, because there is no universally accepted one.

**1.** Based on your experience with infinite sums, formulate what you think is a reasonable definition of the statement

$$\prod_{n=0}^{\infty} a_n = L,$$

where the symbol $\prod$ stands for product, and the $a_n$ and $L$ are complex numbers.

**2.** What does your definition say about the following two examples?

$$\text{(a)} \prod_{n=0}^{\infty} \left(\frac{1}{2^n}\right) \qquad\qquad \text{(b)} \prod_{n=0}^{\infty} (1-2^n)$$

A necessary condition for convergence of an infinite sum $\sum b_n$ is that $b_n \to 0$. The analogous statement for infinite products would be that a necessary condition for convergence of $\prod a_n$ is that $a_n \to 1$.

**3.** Is this a reasonable condition? Is it in accord with your definition of convergence of infinite products?

Since the logarithm of a product is the sum of the logarithms, it is sometimes said that $\prod a_n$ converges if and only if $\sum \log a_n$ converges.

**4.** Is this a reasonable condition? Is it in accord with your definition of convergence of infinite products?

**5.** Finally, reformulate your definition of convergence of infinite products, taking account of the issues developed in parts 2–4.