Exercise on simple connectivity

Consider a square  and an annulus  in the plane. Of course a square is simply connected, and an annulus is not. For each of the following properties, can you verify explicitly—without simply quoting a theorem—that a square has the property, and an annulus lacks the property?

Topological properties equivalent to simple connectivity

1. The complement with respect to the Riemann sphere is connected.
2. The fundamental group is trivial.
3. Every closed curve has zero winding number about every point in the complement.
4. The domain is homeomorphic to the open unit disc.

Analytic properties equivalent to simple connectivity

5. Every holomorphic function has a holomorphic anti-derivative.
6. Every nowhere zero holomorphic function has a holomorphic logarithm.
7. Every nowhere zero holomorphic function has a holomorphic square root.
8. Every harmonic function is the real part of a holomorphic function.
9. Every holomorphic function can be approximated uniformly on compact sets by polynomials.