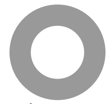
# Exercise on simple connectivity

Consider a square  and an annulus  in the plane. Of course a square is simply connected, and an annulus is not. For each of the following properties, can you verify explicitly—without simply quoting a theorem—that a square has the property, and an annulus lacks the property?

## Topological properties equivalent to simple connectivity

1. The complement of the domain with respect to the Riemann sphere is connected.

2. The fundamental group $\pi_1$ of the domain is trivial. (Every closed curve in the domain is homotopic to a constant curve.)

3. For every piecewise $C^1$ closed curve in the domain and every point in the complement of the domain, the winding number of the curve about the point is equal to 0. (Every piecewise $C^1$ closed curve in the domain is "homologous to zero".)

4. The domain is homeomorphic to the open unit disc.

## Analytic properties equivalent to simple connectivity

5. Every holomorphic function on the domain has a holomorphic antiderivative on the domain.

6. Every holomorphic function on the domain that does not take the value 0 has a holomorphic logarithm on the domain.

7. Every holomorphic function on the domain that does not take the value 0 has a holomorphic square root on the domain.

8. Every harmonic function on the domain is the real part of a holomorphic function on the domain.

9. Every holomorphic function on the domain can be approximated uniformly on compact subsets of the domain by holomorphic polynomials.