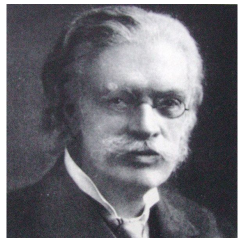
Theory of Functions of a Complex Variable II

Part A

Pick *three* of the following mathematicians, and state a theorem from this course that is named after that person.

Jacques Hadamard
(1865–1963)



G. Mittag-Leffler
(1846–1927)

Paul Montel
(1876–1975)

Émile Picard
(1856–1941)

Carl Runge
(1856–1927)

Part B

Pick *three* of the following items. For each item, either construct an example satisfying the stated conditions or prove that no example exists (whichever is appropriate).

1. An entire function of finite order whose genus does not equal the order.
2. A power series $\sum_{n=0}^{\infty} c_n z^n$ having radius of convergence equal to 1 and such that for every positive integer k, the polynomial $\sum_{n=0}^{k} c_n z^n$ has no zeroes when $|z| < 1$.
3. A holomorphic function f on $\mathbb{C} \setminus \{0\}$ having an essential singularity at 0 and such that $|f(z)| < 1$ in the half-plane where $\operatorname{Re} z < 0$.
4. A sequence $\{u_n\}$ of harmonic functions on the unit disk such that $-1 < u_n(z) < 1$ for every positive integer n and every point z in the unit disk, yet the sequence $\{u_n\}$ has no subsequence that converges uniformly on compact subsets of the unit disk.
5. A holomorphic function on $\mathbb{C} \setminus \mathbb{Z}$, the complex plane with the integers deleted, that has an essential singularity at each integer.