Theorems

State a theorem from this course named for a mathematician whose name starts with the letter

(1) M,

(2) P,

(3) R,

(4) S,

(5) W.

Examples

For each of the following items, either supply an example of a function with the indicated properties, or prove that no such function can exist.

(6) A linear fractional transformation (Möbius transformation) that fixes exactly one point of $\mathbb{C} \cup \{\infty\}$ (the extended complex numbers).

(7) A biholomorphic mapping from $\mathbb{C} \setminus \{ z \in \mathbb{C} : |z| \leq 1 \}$ (the complement of the closed unit disk) onto $\mathbb{C} \setminus \{0\}$ (the punctured plane).

(8) A nonconstant holomorphic function f on $\{ z \in \mathbb{C} : |z| < 1 \}$ (the unit disk) such that the sequence of iterates $f \circ f$, $f \circ f \circ f$, ... converges normally to a constant function.

(9) A holomorphic function on $\{ z \in \mathbb{C} : \operatorname{Im}(z) > 0 \}$ (the upper half-plane) that is *not* the normal limit of a sequence of polynomials.

(10) A nonpolynomial entire function f such that $\lim_{r \to \infty} |f(re^{i\theta})| = \infty$ for every angle θ.