Final Examination

Part A

Some of the mathematicians whose theorems figured in the course are the following.

- Jacques Hadamard (1865–1963)
- Gösta Mittag-Leffler (1846–1927)
- Paul Montel (1876–1975)
- Émile Picard (1856–1941)
- Hermann Amandus Schwarz (1843–1921)

For each of the following conditions, state a theorem from the course named after a mathematician on this list who satisfies the condition.

1. The year of death has no single-digit prime factor.
2. The person is different from the one you chose in question 1, and the person lived at least twice as long as Bernhard Riemann (17 September 1826 – 20 July 1866).
3. The person is different from the ones you chose in questions 1 and 2, and the name does not contain the letter "i."

Part B

Pick *three* of the following items. For each item, either construct an example satisfying the stated conditions or prove that no example exists (whichever is appropriate).

1. A holomorphic function on the unit disk having bounded real part but unbounded imaginary part.
2. A sequence of holomorphic functions mapping the upper half-plane into itself and converging pointwise, but not converging uniformly on compact subsets.
3. A real-valued harmonic function $u(x, y)$ defined on the upper half-plane such that $\lim_{y\to 0^+} u(x, y) = 1$ when $x > 0$ and $\lim_{y\to 0^+} u(x, y) = -1$ when $x < 0$.
4. A holomorphic function that maps the once-punctured plane $\mathbb{C} \setminus \{0\}$ surjectively to the twice-punctured plane $\mathbb{C} \setminus \{0, 1\}$.
5. A sequence (f_n) of entire functions converging uniformly on compact sets to an entire function g such that the derivative g' is never equal to zero, yet f_n' has zeroes for every n.