INTEGRATING COMPUTER TECHNOLOGY INTO THE UNIVERSITY MATHEMATICS CURRICULUM

HAROLD P. BOAS

1. Introduction

Can your mathematics students solve the following three problems?

1. Sketch the surface in three dimensions defined by the equation $z = \sin(x \sin(xy))$.
2. Evaluate the integral $\displaystyle\int_0^\infty \frac{\cos(ax)}{(1+x^2)^2}\,dx$, where $a > 0$.
3. Compute the continued fraction expansion of $\sqrt{17/31}$.

Although a first-year university student can easily understand the statements of these problems, to work out the solutions by hand is a formidable task. Try it yourself! (Answers are given later.)

Many undergraduates at my university, however, can quickly produce answers to these problems by sitting down at a computer console and typing a few keystrokes. To illustrate the capabilities of some popular computer mathematics packages, I offer these sample solutions.

1. The following MATLAB instructions display a three-dimensional plot of the graph of the equation $z = \sin(x \sin(xy))$ over the square where $0 \le x \le 3$ and $0 \le y \le 3$. (This code includes a request for greyscale rendering, rather than color, so that I can print the figure at the end of this article.)
```
x=0:.1:3; y=0:.1:3; [X,Y]=meshgrid(x,y);
Z=sin(X.*sin(X.*Y)); surfl(X,Y,Z);
colormap(gray); shading interp; view(45,45);
```
2. Mathematica evaluates $\displaystyle\int_0^\infty \frac{\cos(ax)}{(1+x^2)^2}\,dx$ in closed form as a function of the positive parameter a via the command
```
Integrate[Cos[a*x]/(1+x^2)^2, {x, 0, Infinity},
          Assumptions->(a>0)]
```
3. Maple supplies the periodic continued fraction expansion of the quadratic surd $\sqrt{17/31}$ in response to the request
```
numtheory[cfrac](sqrt(17/31), periodic);
```

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2. Computers and pedagogy

The power of modern computer mathematics software has great potential for enhancing the education of students. The computer continues to change what and how I teach and what and how my students learn. Here are some examples of ways in which I have used computer software to help students understand mathematical concepts.

2.1. **Classroom demonstrations.** Geometric visualization is a particularly valuable use of the computer. Starting about ten years ago, my department provided convenient access to computing facilities with which I could easily create geometric figures without first having to learn sophisticated programming techniques. I began to prepare illustrations to show to my classes.

For example, when I was teaching three-dimensional analytic geometry, I would prepare a picture of a hyperboloid of one sheet. When I was discussing Taylor polynomials, I would prepare a plot showing how the polynomials of progressively higher degrees approximate the original function. When I was lecturing on heat flow in a cylinder, I would prepare graphs of Bessel functions. I would print the examples on paper, copy the figures onto transparent plastic foils, take the transparencies to class, and display them using a standard overhead projector.

Subsequently, my department invested in more elaborate computing equipment, and I began to replace static illustrations with dynamic ones. By planning carefully in advance, I could take a laptop computer to class and display an animation of partial sums of a Fourier series converging; the students would get excited at *seeing* the Gibbs phenomenon materialize on the screen. I could show an animation in color of upper and lower sums converging to the area under a curve.

Currently, many of the larger lecture halls at my university have computer terminals that are permanently installed and connected to color overhead projection units. In such a room, I can give a spontaneous, interactive presentation. I can display a direction field plot for a differential equation, superimpose integral curves, change the equation in response to students' questions, and have the computer solve the equation either analytically or numerically. I can access the Internet to show the class a picture of Riemann or an illustration of the Borromean rings or a Java applet implementing Conway's game of life.

2.2. **Student projects.** Since mathematics is a participatory activity, not a spectator sport, students should also make their own personal mathematical explorations on the computer. At my institution, all students have access to up-to-date computing facilities, and the students routinely use computers in many of their classes (not just their scientific classes).

Indeed, students pay a fee of approximately $80 each term to support the university's "open-access computer laboratories"; these facilities provide the students with access to electronic mail, the Internet, word processing, spreadsheets, and so forth. Many students own personal computers, and the dormitories are wired for high-speed Internet connections. In addition, the mathematics department maintains extensive student computing laboratories funded by a special fee that mathematics students pay during their first four terms.

Consequently, I feel that I have not only the license but even the obligation to give my students some assignments that require computer use. For example, I can ask my students to sketch by hand the graph of the Γ function on the real axis by applying their theoretical knowledge, and then to compare their result with an accurate plot rendered by the computer. I can ask them to compute both the inverse of the 5×5 Hilbert matrix and the inverse of a three-digit numerical approximation of the matrix, and to provide a theoretical explanation of the disparity in the answers. I can ask them to run numerical simulations to compare the accuracy of the Euler method and the Runge-Kutta method for solving differential equations.

Mathematics today has a strong experimental component, a point that we should not hide from our students. Indeed, my graduate students use the computer extensively to test research questions and to help discover proofs. I have found the computer helpful in my own theoretical work. For example, I recently encountered the equation $\sum_{k=1}^{\infty} k^k x^k / k! = 1/2$. The computer solved this equation numerically, and the Inverse Symbolic Calculator [4] on the World-Wide Web told me that the numerical approximation is apparently the number $x = 1/(3e^{1/3})$. This computer-generated conjecture gave me the curiosity and the confidence to seek a rigorous derivation of the solution, which in due course I found.

3. Implementation

The main obstacle to using computer technology in education is financial: developing the necessary infrastructure can be expensive. In my department, we use Maple rather than Mathematica not because Maple is a superior system, but because the university was able to obtain a campus site license for Maple at a reasonable cost.

As technology continues to advance, sophisticated mathematical programs are becoming less expensive. At my university, students who have already invested in their own personal computer can buy their own copy of Maple for $100. For less than $200, on the other hand, students without a computer can buy a hand-held calculator that has numerical, graphical, and symbolic computational capabilities only a little inferior to those of Maple and Mathematica. A conceivable low-budget solution to providing students access to computational tools is to equip the students with calculators like the Hewlett-Packard HP49G [3] or the Texas Instruments TI89 [10].

My historical experience may be a good model for implementing computer technology at other institutions. First it is necessary to make the hardware and the software available to the faculty. Once the teachers gain familiarity with the capabilities of the computer and see the pedagogical potential, then it is necessary to provide computer access to the students.

Both Maple and Mathematica are powerful, all-purpose mathematics programs that can be used effectively by teachers and by students. MATLAB is another good tool, popular with engineers, but it is not quite as versatile as Maple and Mathematica.

Of course, one has to invest some effort to learn how to use these tools properly. Fortunately, the Maple, Mathematica, and MATLAB programs all contain introductory tutorials and extensive online help. Moreover, good beginner's manuals are available, e.g., [1, 2, 7, 9, 11]. Many other useful books are listed at the programs' Web sites [5, 6, 8].

Undergraduate students at my university develop facility with Maple as an adjunct to their traditional mathematics classes. The calculus course allocates one hour per week for a meeting in the computer laboratory, where students learn about Maple and work on computer projects related to the theoretical topics covered during the lectures. By the end of the first year, students can use Maple to plot graphs, to perform numerical calculations to an arbitrary degree of accuracy, to manipulate algebraic expressions, and to differentiate and integrate elementary functions symbolically. In the second year, they may learn how to use Maple for problems in multidimensional calculus and in linear algebra, and they become familiar with how Maple can solve ordinary differential equations analytically, numerically, and graphically. By this time, students need to be given only hints about going further with Maple; indeed, the students often are more comfortable using the computer than the faculty are.

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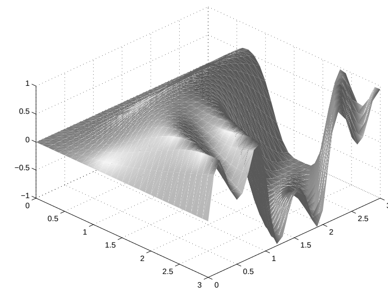

4. Answers

Could you solve the three problems stated in the introduction? The figure shows the periodic continued fraction that Maple found (where $[2, 1, 5, 1, 5, 1, 2, 2]$ is the repeating group) and the plot that MATLAB produced. Mathematica obtained the value $\pi(1 + a)e^{-a}/4$ for the improper integral.

5. Conclusion

Computer technology has had a dramatic impact on mathematics. G. H. Hardy could once characterize number theory as a pure subject with no military use, but cryptography has made number theory today an applied subject of immense interest to government intelligence agencies. Computer-generated images of fractals have made mathematics into a modern art form.

We teachers of mathematics had better adapt to the computer revolution or risk becoming obsolete. I have indicated in this article some of my own efforts to integrate the computer into the university mathematics curriculum. I hope that these models will be useful to other educators.

It is fascinating to speculate about what that prodigious calculator Ramanujan might have discovered if powerful computing machines had been available in his time. Perhaps even now in some small village there is a new Ramanujan growing up who one day will become inspired to pursue mathematical research by seeing fantastic formulas scrolling down a computer screen. I hope so.

References

[1] Jerry Glynn and Theodore Gray, *The Beginner's Guide to Mathematica*, Cambridge University Press, 2000.
[2] K. M. Heal and others, *Maple V Learning Guide*, Springer-Verlag, 1998.
[3] See `http://www.hp.com/calculators/` on the World-Wide Web for information about Hewlett-Packard calculators.
[4] `http://www.cecm.sfu.ca/projects/ISC/` is the World-Wide Web home page of the Inverse Symbolic Calculator.
[5] `http://www.maplesoft.com/` is the World-Wide Web home page of Maple.
[6] `http://www.wri.com/` is the World-Wide Web home page of Mathematica.
[7] The MathWorks, Inc., *The Student Edition of MATLAB 5: User's Guide*, Prentice Hall, 1997.
[8] `http://www.mathworks.com/` is the World-Wide Web home page of MathWorks, the vendors of MATLAB.
[9] M. B. Monagan and others, *Maple V Programming Guide*, Springer-Verlag, 1998.
[10] See `http://www.ti.com/calc/docs/calchome.html` on the World-Wide Web for information about Texas Instruments calculators.
[11] Bruce F. Torrence and Eve A. Torrence, *The Student's Introduction to Mathematica*, Cambridge University Press, 1999.

Department of Mathematics, Texas A&M University, College Station, TX 77843-3368, USA
E-mail address: `boas@math.tamu.edu`